MTH436 - HOMEWORK 10

Not to be handed in, but you may want to attempt the questions before 3pm on May 8th...

Question 1. For the given sequences f_n, decide if the series $\sum f_n$ converges uniformly (here, $a > 0$).

(i) $f_n\colon \mathbb{R} \to \mathbb{R}$ given by $f_n(x) = \frac{1}{n^2+x^2}$.
(ii) $f_n\colon [-a, a] \to \mathbb{R}$ given by $f_n(x) = \frac{x^2+n}{x^2+n^3}$.
(iii) $f_n\colon \mathbb{R} \to \mathbb{R}$ given by $f_n(x) = \sin(x/n^2)$.
(iv) $f_n\colon \mathbb{R} \to \mathbb{R}$ given by $f_n(x) = \frac{x^n}{x^n+1}$.
(v) $f_n\colon \mathbb{R} - \{0\} \to \mathbb{R}$ given by $f_n(x) = \frac{1}{n^2x^2}$.

Question 2. Compute the Radius of convergence for the following power series.

(i) $\sum \frac{2+(-1)^n}{3^n} x^n$.
(ii) $\sum x^{n!}$.
(iii) $\sum a_n x^n$ where

$$a_n = \begin{cases} n & \text{if } n = 3k \text{ for some } k \in \mathbb{N} \cup \{0\}, \\ 0 & \text{if } n = 3k+1 \text{ for some } k \in \mathbb{N} \cup \{0\}, \\ 4^n & \text{if } n = 3k+2 \text{ for some } k \in \mathbb{N} \cup \{0\}. \end{cases}$$

(iv) $\sum a_n x^n$ where $0 < r \leq |a_n| \leq s$ for all n.
(v) $\sum \frac{\sin(n)}{n} x^n$.
(vi) $\sum a_n x^n$ where $a_n = 1$ if $n = m!$ for some n and $a_n = 0$ otherwise.
(vii) $\sum \frac{x^n}{\log n}$.
(viii) $\sum \frac{n!}{n^n} x^n$.

Question 3. Show that if f is an odd function and $f(x) = \sum a_n x^n$ then $a_{2k} = 0$ for all $k \in \mathbb{N} \cup \{0\}$.