MTH307 - HOMEWORK 8A

No submitted work this week.

A. Warm-up Questions

Question A.1. Find inverses of the following functions.

(i) $f\colon \mathbb{Z} \to \mathbb{Z}$, $f(x) = x + 2$.
(ii) $f\colon \mathcal{P}(\mathbb{R}) \to \mathcal{P}(\mathbb{R})$, $f(X) = \mathbb{R} - X$.
(iii) $f\colon \mathbb{R} \to (1, \infty)$, $f(x) = 2^{x^3} + 1$.
(iv) $f\colon \mathbb{R} \to \mathbb{R}$, $f(x) = ax + b$ where $a, b \in \mathbb{R}$ and $a \neq 0$.

Question A.2. Decide if the following functions are bijections, and if they are, find a formula for the inverse.

(i) $H\colon \mathbb{R}^2 \to \mathbb{R}^2$, $H(x, y) = (x^2 - y, x)$.
(ii) $f\colon \mathbb{R}^2 \to \mathbb{R}^2$, $f(x, y) = (x + y, x - y)$.
(iii) $F\colon \mathbb{Z}^2 \to \mathbb{Z}^2$, $F(m, n) = (m + 2n, 2m + 3n)$.
(iv) $G\colon \mathbb{R}^2 \to \mathbb{R}^2$, $G(x, y) = (3xy, x)$.

Question A.3. Let $f\colon X \to Y$. We say $g\colon X \to Y$ is a *left inverse* of f if $g(f(x)) = x$ for all $x \in X$. We say that $h\colon Y \to X$ is a *right inverse* if $f(h(y)) = y$ for all $y \in Y$.

(i) Show that f has a left inverse if and only if it is injective.
(ii) Show that f has a right inverse if and only if surjective.
(iii) If g is a left inverse of f, is it unique? Prove your answer.
(iv) If h is a right inverse of f, is it unique? Prove your answer.

Question A.4. Let X, Y and Z be sets and $f\colon X \to Y$. Prove that f is surjective if and only if for all functions $h\colon Y \to Z$ and $k\colon Y \to Z$, if we have $h \circ f = k \circ f$ then $h = k$.