

MTH307 - HOMEWORK 3

Solutions to the questions in Section B should be submitted by the start of class on 2/22/18. You may like to remind yourself on the properties of integers and real numbers found on pages 23 and 27 respectively of the textbook.

A. WARM-UP QUESTIONS

Question A.1. Prove the following.

- (i) If x is odd then $x^2 + 3x + 9$ is odd.
- (ii) If $x \in \mathbb{R}$ then $x^2 \geq 0$ (Use cases).
- (iii) If $a \mid b$ and $c \mid d$ then $ac \mid bd$.
- (iv) If $x < 0$ then $x + \frac{1}{x} \leq -2$ (Try working backwards).
- (v) If $0 < a < b$ then $0 < a^2 < b^2$.
- (vi) If x is odd then x^3 is odd.
- (vii) If $x \in \mathbb{R}$ and $x \neq 0$ then there exists a unique $y \in \mathbb{R}$ such that $xy = 1$.

Question A.2. Prove or disprove the following.

- (i) Suppose $a, b, c \in \mathbb{Z}$. If $a \mid bc$ then $a \mid b$ or $a \mid c$.
- (ii) Suppose $a, b, c \in \mathbb{Z}$. If $a \mid (b + c)$ then $a \mid b$ or $a \mid c$.
- (iii) If m and n are even then $4 \mid mn$.

B. SUBMITTED QUESTIONS

Question B.1. Prove or disprove the following.

- (i) If $x \in \mathbb{Z}$ then $5x^2 + 3x + 7$ is odd.
- (ii) If $x > 0$ then $x + \frac{1}{x} \geq 2$.

C. CHALLENGE QUESTIONS

Question C.1. Prove the following.

- (i) If $n^2 \mid n$ then $n = 1$ or $n = 0$ or $n = -1$.
- (ii) Suppose $x, y \in \mathbb{R}$. If $x^2 + 5y = y^2 + 5x$ then $x = y$ or $x + y = 5$.
- (iii) If $a, b, c \in \mathbb{R}$ then $ax^2 + bx + c = 0$ has a (real) solution u if and only if $b^2 - 4ac \geq 0$ and $u = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
- (iv) If m is an odd integer, then there exists $k \in \mathbb{Z}$ such that $m^2 = 8k + 1$.