MTH307 - HOMEWORK 2

Solutions to the questions in Section B should be submitted by the start of class on 2/15/18.

A. Warm-up Questions

Question A.1. Find converses and contrapositives to the following statements (don't worry about what all the words mean!).

(i) If n is prime then n is odd.
(ii) For a function to be integrable, it is necessary that it is continuous.
(iii) A sequence converges whenever it is bounded and monotonic.

Question A.2. Negate the following symbolic sentences.

(i) $\forall x \, \exists y (x = -y)$
(ii) $\exists y \, \forall x (P(x) \vee Q(y))$
(iii) $\forall x \, \forall y \, \exists z \, (xy < z)$
(iv) $\exists x \, \forall y \, \forall z \, (P(x) \Rightarrow (Q(y) \wedge R(z)))$

Question A.3. Convert the followings into symbolic logic (do not worry about the veracity of these statements!).

(i) If $ab = 0$ then $a = 0$ or $b = 0$.
(ii) For all prime numbers p, there is a prime number q such that $q > p$.
(iii) The number x is positive but the number $x + y$ is not positive.

Question A.4. Prove the following.

(i) If n is odd then n^2 is odd.
(ii) If m is odd and n is odd then $m + n$ is even.
(iii) The product of an odd number and an even number is even.

B. Submitted Questions

Question B.1. Find a useful denial of the following (this is the definition that $a_n \to 0$).

- For every $\varepsilon > 0$ there exists a natural number N such that $|a_n| < \varepsilon$ whenever $n > N$.

Question B.2. Prove the following.

- The product of two odd numbers is odd.

C. Challenge Questions

Question C.1. Find useful denials of the following.

(i) There exists a real number a such that $a + x = x$ for all real numbers x.
(ii) A matrix A is invertible if and only if $\det(A) \neq 0$.
(iii) If $x > 1$ or $x < -2$ then $f(x) \neq 3$.

Question C.2. Prove the following. You may use the Basic Properties of Real numbers on p27.

(i) For all $n \in \mathbb{N}$, the integer $6n - 11$ is odd.
(ii) For all real numbers a and b, $ab = 0$ if and only if $a = 0$ or $b = 0$.
(iii) If $0 < x < y$ then $0 < x^2 < y^2$.
(iv) For all $n \in \mathbb{Z}$ there exists $m \in \mathbb{Z}$ such that $m + n < 0$.