MTH307 - HOMEWORK 1

Solutions to the questions in Section B should be submitted by the start of class on 2/8/18.

A. Warm-up Questions

Question A.1. Show that the following pairs of statements are logically equivalent.

(i) P and $\neg(\neg P)$.
(ii) $P \vee Q$ and $Q \vee P$.
(iii) P and $(\neg P) \Rightarrow (Q \wedge (\neg Q))$
(iv) $(P \wedge Q) \vee R$ and $(P \vee R) \wedge (Q \vee R)$
(v) $P \wedge (Q \wedge R)$ and $(P \wedge Q) \wedge R$
(vi) $P \Rightarrow Q$ and $(\neg Q) \Rightarrow (\neg P)$

Question A.2. Prove DeMorgan's Laws. That is, show the following are logically equivalent.

(i) $\neg(P \wedge Q)$ and $(\neg P) \vee (\neg Q)$.
(ii) $\neg(P \vee Q)$ and $(\neg P) \wedge (\neg Q)$.

Question A.3. A *tautology* is a statement that is true no matter the truth values of the statement letters that occur in it. A *contradiction* is a statement that is false no matter the truth values of the statement letters that occur in it. Show the following.

(i) $P \vee (\neg P)$ is a tautology.
(ii) $P \Rightarrow P$ is a tautology.
(iii) $P \wedge (\neg P)$ is a contradiction.
(iv) $P \Leftrightarrow (\neg P)$ is a contradiction.

B. Submitted Questions

Question B.1. Decide if the following pairs of statement are logically equivalent and justify your answer.

(i) $P \Rightarrow Q$ and $(\neg P) \vee Q$.
(ii) $P \vee (Q \wedge R)$ and $(P \vee Q) \wedge R$.

Question B.2. A *contradiction* is a statement that is false no matter the truth values of the statement letters that occur in it. Decide if the following statements tautologies, contradictions or neither and justify your answer.

(i) $P \Rightarrow (Q \Rightarrow P)$
(ii) $(P \wedge (\neg Q)) \vee ((\neg P) \wedge Q))$.

C. Challenge Questions

Question C.1. Decide if the following are logically equivalent.

(i) $(\neg P) \Leftrightarrow Q$ and $(P \Rightarrow \neg Q) \wedge (\neg Q \Rightarrow P)$.
(ii) $P \Rightarrow (Q \Rightarrow R)$ and $(P \Rightarrow Q) \Rightarrow R$.
(iii) $(P \wedge (\neg Q)) \vee (Q \wedge (\neg P))$ and $(P \vee Q) \wedge (\neg(P \wedge Q))$.
(iv) $(P \Rightarrow R) \wedge (Q \Rightarrow R)$ and $(P \wedge Q) \Rightarrow R$.

Question C.2. Define the logical connective $*$ by the formula $P * Q \equiv (\neg P) \wedge (\neg Q)$.

(i) Show that $\neg P \equiv P * P$
(ii) Show that $P \wedge Q \equiv (P * P) * (Q * Q)$
(iii) Show that $P \vee Q \equiv (P * Q) * (P * Q)$
(iv) Show that $P \Rightarrow Q \equiv ((P*P)*Q)*((P*P)*Q)$
(v) Is it true that $(P*Q)*R \equiv P*(Q*R)$?

Conclude that we can write all logical statements just using the operator $*$. Also conclude that this may be more trouble than its worth.