## MTH243

## EXAMPLES OF FUNCTIONS OF TWO VARIABLES

Here are some examples of functions of two variables.
Example 1 (Graphical). When we look at a map with isotherms on it, we are in fact looking at a function of two variables (the "North" longitude and the "East" Latitude). This is a graphical representation of a function.


The maps shows the temperature at a given time. We may use the picture to estimate the temperature in Rhode Island as $\qquad$
Example 2 (Numerical). Sometimes a function is given numerically - perhaps in a table of values. For example, suppose you want to send some freight. The cost of the delivery will be a function of the weight of the goods and the distance you want them transported. Below is a table showing the costs involved.

|  | Weight (lbs) |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 10 | 20 | 30 | 40 | 50 |  |  |
|  | 100 | 55 | 75 | 92 | 105 | 112 |  |
|  | 200 | 90 | 126 | 143 | 152 | 158 |  |
| Distance <br> (miles) | 300 | 120 | 151 | 169 | 181 | 186 |  |
|  | 400 | 144 | 168 | 178 | 188 | 194 |  |
| 500 | 160 | 172 | 184 | 193 | 201 |  |  |

This allows us to read off prices from the table. For example, the cost to carry $301 b s$ of freight a distance of 200 miles is $\qquad$
Example 3 (Algebraic). Typically in a calculus class, we are more concerned with functions given by an algebraic formula. In many cases, we may try to model graphical or numerical examples like those above, so that we may use the full power of calculus to study them. As a simple example, the area of a rectangle of height $h$ and width $w$ is given by the formula

$$
A(h, w)=h w
$$

and the perimeter is given by the formula

$$
P(h, w)=2 h+2 w .
$$

Thus if we have a rectangle of height 3 and width 2 , we can compute the area as $A(3,2)=$ and the perimeter $P(3,2)=$

Granted, this is a very simple example!
In the main, we will be concerned with the third type of function, since these are the ones which allow us to use the full power of calculus!

