Clustering in the Mating Operation

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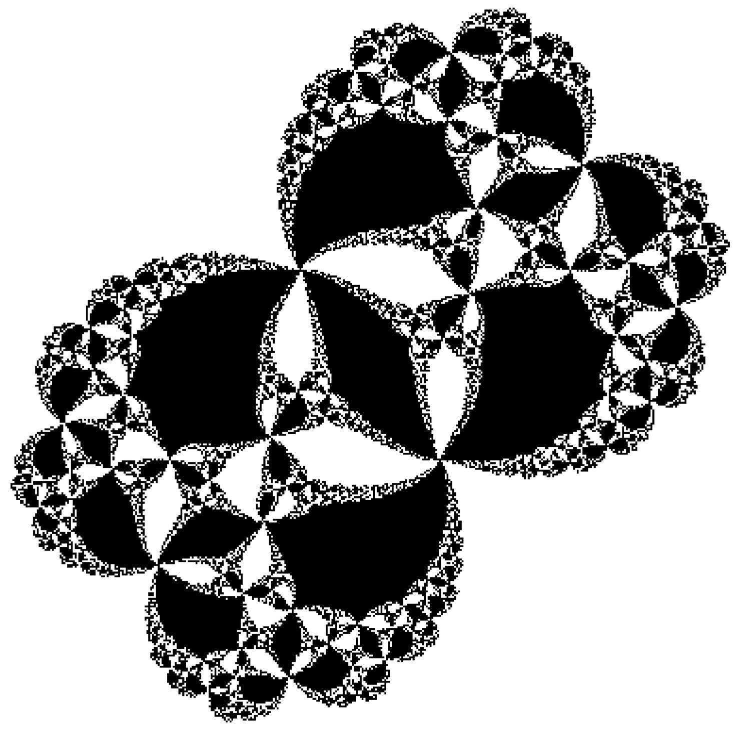
What is clustering?

A cluster point is a (periodic) point which lies on the intersection of the boundaries of the critical orbit components of a rational map F.



The notion of clustering can be observed when considering degree 2 rational maps of type D, when the cluster point belongs to the boundary of components of critical orbit points of both critical points. In particular, we can look at type D maps with clustering that are constructed by the mating of two quadratic polynomials. For example, in the diagram above, the map is constructed by mating the rabbit polynomial with the airplane polynomial.

Combinatorics of clusters

We will restrict ourselves to the case where the rational maps are post-critically finite. We can study the combinatorial data at a cluster point. There are two relevant pieces of data.

- Combinatorial rotation number, ρ
- Critical displacement, δ

The critical displacement measures how far the critical points are from each other, measured in terms of the number of components in the cluster that lie between them. To each cluster we can define an ordered pair (ρ, δ) called the combinatorial data of the cluster.

Theorem. *The combinatorial data uniquely (in the sense of Thurston) defines the rational map in the case where the cluster is fixed or in the case where the map is quadratic and the cluster has period two.*

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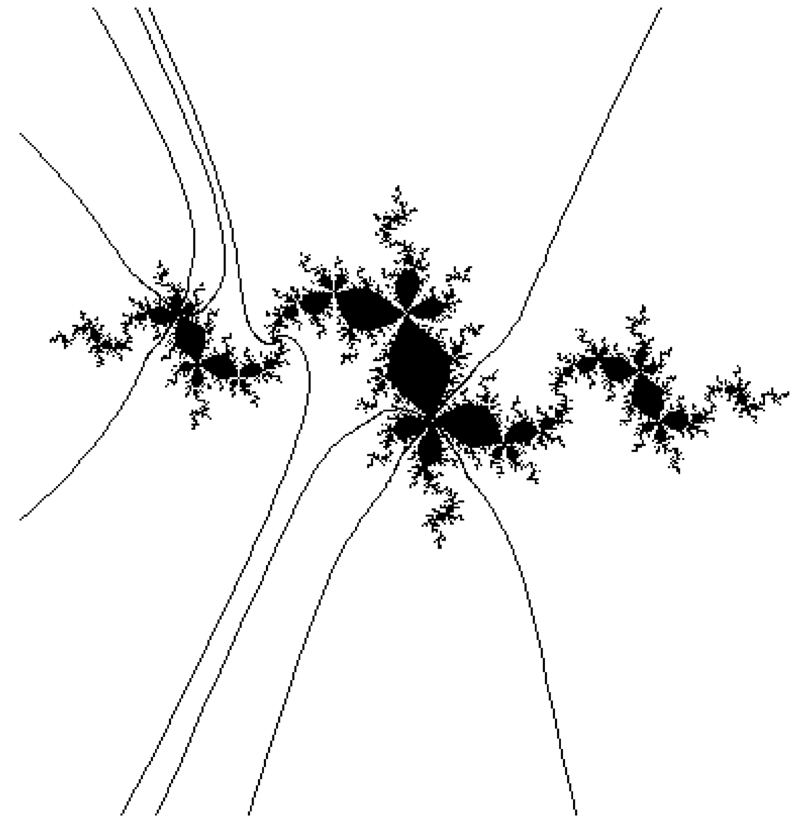
Clusters arising from matings

An obvious way to construct rational maps with cluster points is by mating two polynomials. The fixed case is very simple.

Theorem. *If f_1 and f_2 mate to produce a rational map with a fixed cluster then precisely one of them is a rabbit.*

In the two cluster case, the obvious way of constructing the rational map (in light of the above result) is to take the "double-rabbit" (a map bifurcating off of the period 2 hyperbolic component) and "fill in the gaps" between the critical orbit components.



However, we notice there is a second period $2n$ map which has the same angles landing at its repelling period two cycle.

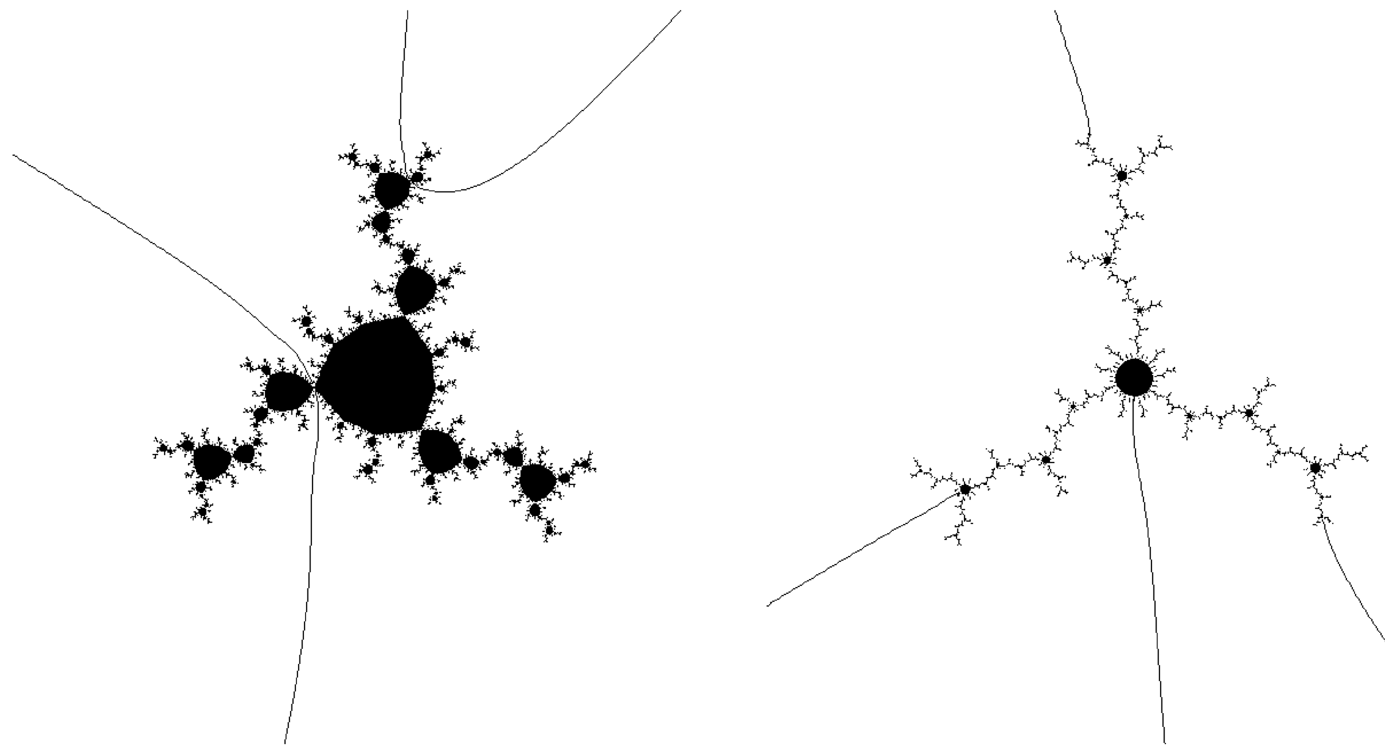
Theorem. *If f_1 and f_2 mate to produce a map with a period 2 cluster cycle then precisely one of them is a tuned rabbit or the secondary period $2n$ component in the wake of the period $2n$ tuned rabbit.*

Theorem. *All rational maps with fixed cluster points or period two cluster points are matings.*

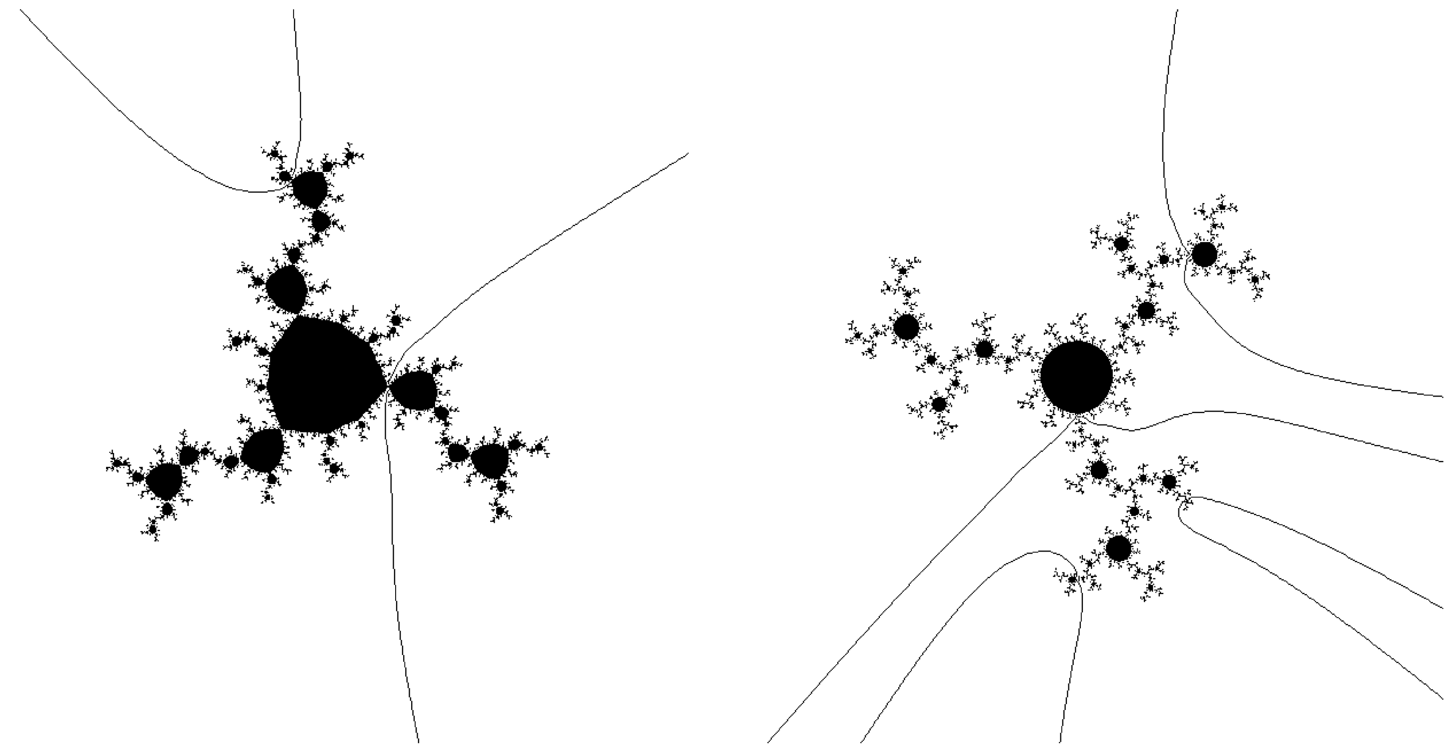
Considerations in the period two higher degree case

The combinatorial data (ρ, δ) is not enough to classify the rational map in the sense of Thurston when the rational map is of degree $d \geq 3$ and the clusters have period two.

The following pair of degree 3 polynomials mate to create a rational map F_1 with *(intrinsic)* combinatorial data $(1/2, 3)$.



The following pair of degree 3 polynomials mate to create a rational map F_2 which also has (intrinsic) combinatorial data $(1/2, 3)$.



In both cases the first polynomial is a "double-rabbit". We refer to the first mating as a *back-door* mating: the ray class in the mating which becomes the cluster cycle contains the non-principal root points of the critical orbit components of the second polynomial. F_1 and F_2 are **not** equivalent in the sense of Thurston. What is the missing piece of extra combinatorial data that we need in this case?

Conjecture. *The extra piece of data can be characterised by consideration of the fixed points of the rational maps.*

Details at http://arxiv.org/abs/1108.5324 and http://arxiv.org/abs/1108.4808 or http://wrap.warwick.ac.uk/35776/