Integral and matrix inequalities associated with graphs

Alexander Sidorenko
sidorenko.ny@gmail.com

Let $\mathcal{G}$ denote the space of bounded symmetric measurable functions on $[0,1]^2$. For a graph G with vertices $\{v_1, v_2, \ldots, v_n\}$ and edge set E, we define a functional $t_G : \mathcal{G} \to \mathbb{R}$ as

$$t_G(f) \;=\; \int_{[0,1]^n} \prod_{\{v_i,v_j\}\in E} f(x_i, x_j)\, dx_1 dx_2 \cdots dx_n \;.$$

We study inequalities of such types as

- majorization: $t_G \;\le\; t_H$;
- positivity: $t_G \;\ge\; 0$;
- convexity: $2t_G(f) \;\le\; t_G(f+h) + t_G(f-h)$.