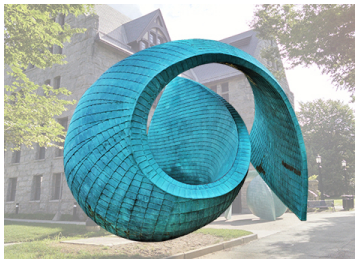

UNIVERSITY OF RHODE ISLAND

Department of Mathematics
and Applied Mathematical Sciences

Applied Mathematics and Scientific Computing Seminar

Location: Lippitt Hall 205
Time: Monday, April 29, 2024, 1:00pm
(refreshments at 12:55 p.m.)

GMRES – An Iterative Method for Solving Large Linear Systems

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Abstract: In this talk we will investigate the Generalized Minimal Residuals (GMRES) algorithm for finding an approximate solution to a linear system $Ax = b$, where A is an $n \times n$ matrix and n is generally considered to be quite large. GMRES is an iterative algorithm developed by Saad and Schultz [1] which can be described very concisely: *approximate the solution of* $Ax = b$ *by a vector* $\tilde{x} \in x_0 + \mathcal{K}_m(A, r_0)$*, where* x_0 *is an* initial guess *(often the zero vector),* $r_0 = b - Ax_0$ *is the residual, and*

$$\mathcal{K}_m(A,\, r_0) \;:=\; \mathtt{span}\,\{r_0,\, Ar_0,\, A^2 r_0,\, \ldots,\, A^{m-1} r_0\}\,,$$

is the Krylov subspace with $m << n$*.*

We will begin by explaining why iterative methods, such as GMRES, are important for solving large linear systems, i.e., why Gaussian Elimination is not the be-all and end-all. Next we provide necessary background underpinning many iterative algorithms like GMRES, including the Arnoldi algorithm and an introduction to Krylov subspaces. We also discuss several aspects regarding implementation of GMRES and provide several numerical examples showcasing key properties about convergence and accuracy of GMRES. Finally, time permitting, we briefly discuss the limitations of the algorithm and how future work can expand on and/or strengthen the procedure.

Pre-requisite: Exposure to theoretical or numerical linear algebra beyond the first course.

[1] Y. Saad and M.H. Schultz, "GMRES: A generalized minimal residual algorithm for solving nonsymmetric linear systems," *SIAM J. Sci. Stat. Comput.*, pp.856–869, 1986.