Math 562 Homework 6

Dr. Mark Comerford

Due Tuesday May 1, 2012

All problems worth 20 points, including bonus problems

1. Contour Integrals

Calculate (using residues!)

(a)
$$\int_0^\infty \frac{\mathrm{d}x}{4+x^2},$$
 (b)

$$\int_0^\infty \frac{\mathrm{d}x}{(4+x^2)^2}.$$

2. More Contour Integrals

Evaluate the following integral:

$$\int_0^{2\pi} \frac{\mathrm{d}t}{1+\cos^2 t}.$$

3. Calculating Certain Infinite Sums

Show that if a is not an integer, then

$$\sum_{n=-\infty}^{\infty} \frac{1}{(a+n)^2} = \frac{\pi^2}{\sin^2 \pi a}.$$

You should explain how you are justified in using any results given in class to get your answer.

Please turn over!

4. *Something a Little Harder

Evaluate the following integral:

$$\int_0^\infty \frac{\mathrm{d}x}{1+x^3}.$$

Hint: The required contour is similar to a semicircle but it is *not* a semicircle. Instead you should choose something which is more sympathetic to the particular integrand in the question!