Chapter 3 Probability Distributions and Probability Densities

§ 3.1 Random Variables

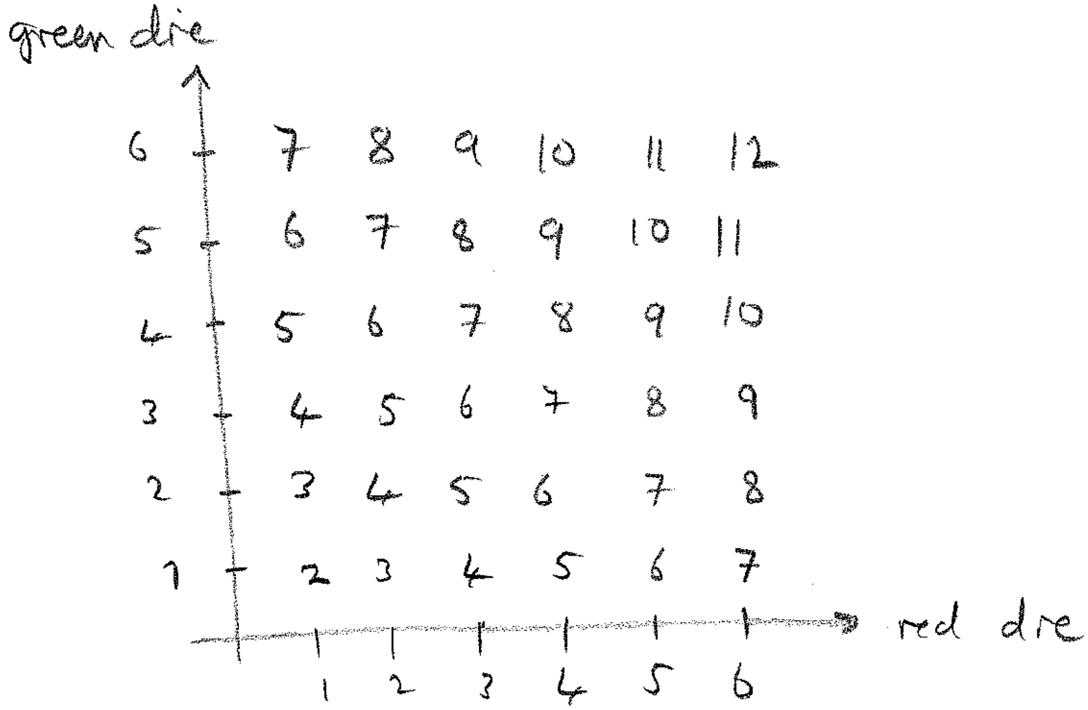
Sometimes we are only interested in certain aspects of the outcome of an experiment. Random variables are used to quantify these aspects.

Ex. Roll two dice, one red one green.

Each point in the sample space has the same prob $\frac{1}{36}$.

An example of a random variable would be the sum of the numbers on the two dice.

The assignment of values to sample points looks like



<u>Defn 3.1</u> If S is a sample space with a prob. measure, a real-valued function $X : S \longrightarrow \mathbb{R}$ is called a <u>random variable</u> on S.

Q. What's random about this?

A. The probabilities of the events in S. Although $X(x)$ is fixed once we know x, just which value of x in S gets picked <u>is</u> random.

In the previous ex. X had 9 as one of the possible values.

The set $X = 9$ or $\{x \in S \mid X(x) = 9\}$ correspond to the event $\{(6,3), (5,4), (4,5), (3,6)\}$.

Ex. Two socks are selected at random & removed from a drawer which has 5 brown socks and 3 green socks. List the elements of the sample space, the corresponding probs. and the corresponding values w of the random variable W, where W is the number of brown socks selected.

Soln. Let B = brown, G = green.

Then $S = \{BB, BG, GB, GG\}$

and $P(BB) = \frac{5}{8} \times \frac{4}{7} = \frac{5}{14}$, $P(BG) = \frac{5}{8} \times \frac{3}{7} = \frac{15}{56}$,

$P(GB) = \frac{3}{8} \times \frac{5}{7} = \frac{15}{56}$, $P(GG) = \frac{3}{8} \cdot \frac{2}{7} = \frac{3}{28}$

For the r.v. W we have the table.

Element of S	Prob.	W
B B	$5/14$	2
B G	$15/56$	1
G B	$15/56$	1
G G	$3/28$	0

e.g. $P(W=2) = 5/14$.

Ex. Balanced Coin is tossed 4 times.

X = total no. of heads.

Soln. All 16 samp pts equally likely with prob $1/16$.

Element of Sample Space	Probability	x
HHHH	$\frac{1}{16}$	4
HHHT	$\frac{1}{16}$	3
HHTH	$\frac{1}{16}$	3
HHTT	$\frac{1}{16}$	2
HTHH	$\frac{1}{16}$	3
HTHT	$\frac{1}{16}$	2
HTTH	$\frac{1}{16}$	2
HTTT	$\frac{1}{16}$	1
THHH	$\frac{1}{16}$	3
THHT	$\frac{1}{16}$	2
THTH	$\frac{1}{16}$	2
THTT	$\frac{1}{16}$	1
TTHH	$\frac{1}{16}$	2
TTHT	$\frac{1}{16}$	1
TTTH	$\frac{1}{16}$	1
TTTT	$\frac{1}{16}$	0

Counting up we can see, for example, that

$$P(X=3) = \frac{4}{16}.$$

All the above are discrete random variables which can only take on finitely or countably infinitely many values. We will meet continuous random variables later.