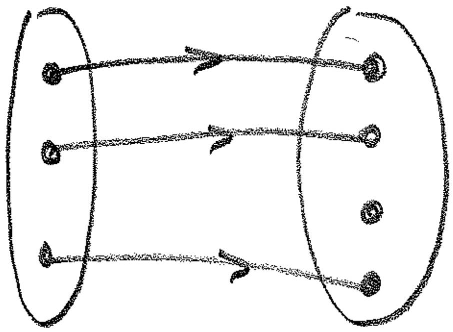
Def^{n}. Let $T : \mathbb{R}^n \to \mathbb{R}^m$.

T is onto (surjective) if each $b \in \mathbb{R}^m$ is the image of at least one $x \in \mathbb{R}^n$;
i.e. $\exists$ at least one $x \in \mathbb{R}^n$ s.t.

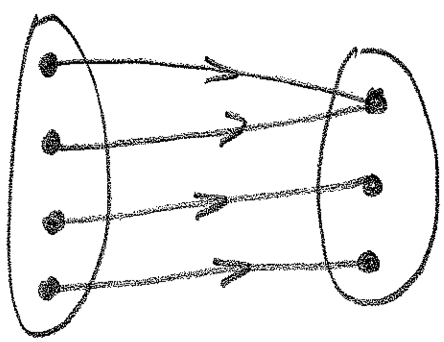
$$Tx = b.$$

T is one-to-one (injective) if each $b \in \mathbb{R}^m$ is the image of at most one $x \in \mathbb{R}^n$

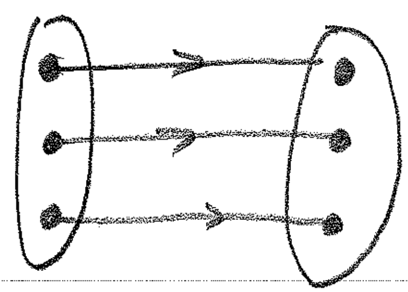
T is bijective if it is injective and surjective.



1-1, not onto



onto, not 1-1



bijective

<u>Ex 4</u> Let T be the linear transformation whose standard matrix is

$$A = \begin{bmatrix} 1 & -4 & 8 & 1 \\ 0 & 2 & -1 & 3 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

Does T map $\mathbb{R}^4$ onto $\mathbb{R}^3$?

Is T a 1-1 mapping?

A is in echelon form & has a pivot position in every row.

By Thm 4 (Section 1.4) $Ax = b$ is consistent (has a sol^n) for each b in $\mathbb{R}^3$. Hence T is onto.

But $Ax = b$ has a free variable (which one) so the sol^n is not unique & so T is not 1-1.

<u>Theorem 11</u> Let $T: \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation. Then T is 1-1 iff the eq^n $Tx = 0$ has only the trivial sol^n.

<u>Theorem 12</u> Let $T: \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation and let A be the standard matrix for T. Then

a. T maps $\mathbb{R}^n$ onto $\mathbb{R}^m$ iff the columns of A span $\mathbb{R}^m$

b. T is 1-1 iff the columns of A are lin. ind.

Ex 5. $T(x_1, x_2) = (3x_1 + x_2, 5x_1 + 7x_2, x_1 + 3x_2)$

$$T(x) = \begin{bmatrix} 3x_1 + x_2 \\ 5x_1 + 7x_2 \\ x_1 + 3x_2 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 5 & 7 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

and so the standard matrix A for T is

$$A = \begin{bmatrix} 3 & 1 \\ 5 & 7 \\ 1 & 3 \end{bmatrix}$$

The cols. of A are l.i (why)
so T is 1-1 by Thm 12

Since A has at most 2 pivot positions and we need 3 for the columns of A to span $\mathbb{R}^3$ (by Thm 4), the columns of A do <u>not</u> span $\mathbb{R}^3$ & so T is not onto by Thm 12.