For All Practical Purposes

Mathematical Literacy in Today's World, 9th ed.

Chapter 21: Saving Models

Section 21.2 Geometric Growth and Compound Interest

James Baglama
Department of Mathematics
University of Rhode Island

Geometric Growth

- Geometric growth is the growth proportional to the amount present (also called exponential growth).

Compound Interest

- Interest paid on both the principal and on the accumulated interest.

10% × \$1000 = 0.10 × \$1000 = \$100 ➔ \$1,100. (end of first year)
10% × \$1100 = 0.10 × \$1100 = \$100 ➔ \$1,210. (end of second year)

Rate per Compounding Period – For a nominal annual rate of interest *r* compounded *m* times per year, the rate per compounding period is:

$$i = {}^{r}/_{m}$$

Compound Interest Formula –

$$A = P\,(1 + i)^{mt}$$

Where:

- A = Amount earned after interest is made
- P = Principal amount
- i = Interest rate per compounding period, which is computed as $i = r / m$
- m = Number of compounding periods
- $n = mt$ = Total number of compounding periods
- t = Time of the loan in years

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Compounding Period

- The amount of time elapsing before interest is paid.

For the examples below *(annual, quarterly, and monthly compounding)*, the amount earned increases when interest is paid more frequently.

Example: Suppose the initial balance is $1000 ($P$ = $1000) and the interest rate is 10% ($r = 0.10$). What is the amount earned in 10 years ($t = 10$) for the following compounding periods, m?

To answer this problem you need to use the following equations:

Rate per compounding period, $i = r / m$

Compound Interest Formula, $A = P(1 + i)^{mt}$

Annual compounding: $i = 0.10$, and $mt = (1)10$ years

$A = \$1000(1 + 0.10)^{10} = \$1000(1.10)^{10} =$ **$2593.74**

Quarterly compounding: $i = 0.10/4 = 0.025$, and $mt = (4)(10) = 40$ quarters

$A = \$1000(1 + 0.025)^{40} = \$1000(1.025)^{40} =$ **$2685.06**

Monthly compounding: $i = 0.10/12 = 0.008333$, and $mt = (12)(10) = 120$mo.

$A = \$1000(1 + 0.10/12)^{120} =$ **$2707.04**

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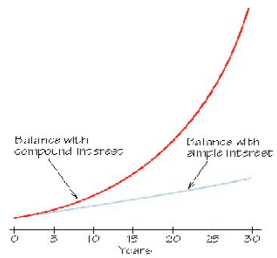
Compound Interest Compared to Simple Interest

- The graph compares the growth of $1000 with compound interest and with simple interest
- r = 10%
- The straight line explains why growth simple interest is also known as linear growth.
- Example of geometric and arithmetic growth:
  Thomas Robert Malthus (1766 –1843), an English demographer and economist, claimed that human population grows geometrically but food supplies grow arithmetically —which he attributed to future problems.

TABLE 21.3 The Growth of $1000: Compound Interest Versus Simple Interest

Years	Amount in Account from Compounded Interest	Amount from Simple Interest
1	1100.00	1100.00
2	1210.00	1200.00
3	1331.00	1300.00
4	1464.10	1400.00
5	1610.51	1500.00
10	2593.74	2000.00
20	6727.50	3000.00
50	117,390.85	6000.00
100	13,780,612.34	11,000.00



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Old Exam Question

If you deposit $2000 at 5% compounded annually, what is the balance after 2 years?

__A. $2200.00

__B. $2205.00

__C. $2297.76

__D. $2300.52

$$\$2000(1+0.05)^2 = \$2205.00$$

Terminology for Interest Rates

A **nominal rate** is any stated rate of interest for a specified length of time, such as a 3% annual interest rate on a saving account or a 1.5% monthly rate on a credit card balance. By itself, such a rate *does not indicate or take into account whether or how often interest is compounded.*

The effective rate is the actual percentage rate of increase for a length of time, *taking into account compounding*.

When stated per year, the effective rate is called the **effective annual rate (EAR)**. For saving, it is also called the **annual percentage yield (APY)** or *annual equivalent yield*.

- Effective Rate
  - The effective rate of interest is:
    - effective rate $= (1+i)^n - 1$ where i=r/m and n=mt.
- Annual Percentage Yield (APY)
  - The amount of interest earned in 1 year with a principal of $1.
  - The annual (i.e. t = 1) effective rate of interest.

$$APY = \left(1+\frac{r}{m}\right)^m - 1$$

- Example: With a nominal rate of 6% compounded monthly, what is the APY?
  - Solution:

$$APY = \left(1+\frac{.06}{12}\right)^{12} - 1 = .0617 = 6.17\%$$

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Old Exam Question

What is the APY for 5.3% compounded quarterly?

$$APY = (1 + 0.053/4)^4 - 1 = 0.0541 = 5.41\%$$