

MATH 215  
Practice

Solve the given systems of equations and represent your solution in parametric vector form  $x = s \cdot u + t \cdot v$ . (section 1.5 pages 45-47).

$$2x_1 - 4x_2 - 4x_3 = 0$$

$$[2 \ -4 \ -4 \ | \ 0] \sim [1 \ -2 \ -2 \ | \ 0]$$

$$x_1 - 2x_2 - 2x_3 = 0$$

$$x_1 = 2x_2 + 2x_3$$

$$x_2 = x_2 \text{ free}$$

$$x_3 = x_3 \text{ free}$$

$$\vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = x_2 \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$$

$$2x_1 - 4x_2 - 4x_3 = 6$$

$$[2 \ -4 \ -4 \ | \ 6] \sim [1 \ -2 \ -2 \ | \ 3]$$

$$x_1 - 2x_2 - 2x_3 = 3$$

$$x_1 = 3 + 2x_2 + 2x_3$$

$$x_2 = x_2$$

$$x_3 = x_3$$

$$\vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix} + x_2 \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$$

Represent the linear system as a vector equation and matrix equation. Solve the system and represent your solution in parametric vector form.

$$x_1 - 2x_2 + 7x_3 = 6$$

$$-2x_1 + 4x_2 + 6x_3 = 8$$

$$3x_1 - 6x_2 - 9x_3 = -12$$

$$\text{V.E. } x_1 \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} + x_2 \begin{pmatrix} -2 \\ 4 \\ -6 \end{pmatrix} + x_3 \begin{pmatrix} 7 \\ 6 \\ -9 \end{pmatrix} = \begin{pmatrix} 6 \\ 8 \\ -12 \end{pmatrix}$$

$$\text{M.E. } \begin{bmatrix} 1 & -2 & 7 \\ -2 & 4 & 6 \\ 3 & -6 & -9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \\ -12 \end{bmatrix}$$

Solve Augmented matrix

$$\begin{bmatrix} 1 & -2 & 7 & | & 6 \\ -2 & 4 & 6 & | & 8 \\ 3 & -6 & -9 & | & -12 \end{bmatrix} \xrightarrow{\substack{2R_1 + R_2 \rightarrow R_2 \\ -3R_1 + R_3 \rightarrow R_3}} \begin{bmatrix} 1 & -2 & 7 & | & 6 \\ 0 & 0 & 20 & | & 20 \\ 0 & 0 & -30 & | & -30 \end{bmatrix} \xrightarrow{60R_2 \rightarrow R_2} \begin{bmatrix} 1 & -2 & 7 & | & 6 \\ 0 & 0 & 1 & | & 1 \\ 0 & 0 & -30 & | & -30 \end{bmatrix} \xrightarrow{\substack{-7R_2 + R_1 \rightarrow R_1 \\ 30R_2 + R_3 \rightarrow R_3}} \begin{bmatrix} 1 & -2 & 0 & | & -1 \\ 0 & 0 & 1 & | & 1 \\ 0 & 0 & 0 & | & 0 \end{bmatrix}$$

Pres

$$\begin{bmatrix} 1 & -2 & 0 & | & -1 \\ 0 & 0 & 1 & | & 1 \\ 0 & 0 & 0 & | & 0 \end{bmatrix}$$

$$\left. \begin{aligned} x_1 - 2x_2 &= -1 \\ x_3 &= 1 \end{aligned} \right\}$$

$$\left. \begin{aligned} x_1 &= -1 + 2x_2 \\ x_2 &= x_2 \text{ free} \\ x_3 &= 1 \end{aligned} \right\}$$

$$\vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -1 + 2x_2 \\ x_2 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} + x_2 \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$$