Represent each linear system as a vector equation and matrix equation.

$$5x_1 - 6x_2 = 4$$
$$3x_1 + 7x_2 = 8$$

V.E. $\quad x_1 \cdot \begin{bmatrix} 5 \\ 3 \end{bmatrix} + x_2 \cdot \begin{bmatrix} -6 \\ 7 \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$

M.E. $\quad \begin{bmatrix} 5 & -6 \\ 3 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$

$$x_1 + 2x_2 + x_3 = 1$$
$$3x_1 + x_2 + 4x_3 = 0$$
$$2x_1 + 2x_2 + 3x_3 = 2$$

V.E. $\quad x_1 \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$

M.E. $\quad \begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & 4 \\ 2 & 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$

Represent each linear system as a vector equation and matrix equation. Then solve the given systems of equations.

$$x_1 + 4x_2 - 2x_3 = 4$$
$$2x_1 + 7x_2 - x_3 = -2$$
$$2x_1 + 9x_2 - 7x_3 = 1$$

V.E. $\quad x_1 \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} 4 \\ 7 \\ 9 \end{bmatrix} + x_3 \begin{bmatrix} -2 \\ -1 \\ -7 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ 1 \end{bmatrix}$

M.E. $\quad \begin{bmatrix} 1 & 4 & -2 \\ 2 & 7 & -1 \\ 2 & 9 & -7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ 1 \end{bmatrix}$

Augmented matrix for solving L.S.

$$\begin{bmatrix} 1 & 4 & -2 & | & 4 \\ 2 & 7 & -1 & | & -2 \\ 2 & 9 & -7 & | & 1 \end{bmatrix}$$

$-2R_1 + R_2 \rightarrow R_2$
$-2R_1 + R_3 \rightarrow R_3$

$$\begin{bmatrix} 1 & 4 & -2 & | & 4 \\ 0 & -1 & 3 & | & -10 \\ 0 & 1 & 3 & | & 1 \end{bmatrix}$$

No solution.

$R_2 + R_3 \rightarrow R_3 \rightarrow \begin{bmatrix} 1 & 4 & -2 & | & 4 \\ 0 & -1 & 3 & | & -10 \\ 0 & 0 & 0 & | & -9 \end{bmatrix}$

$$x_1 - 3x_2 + 2x_3 - x_4 = 8$$
$$3x_1 - 7x_2 \quad + x_4 = 0$$

V.E. $\quad x_1 \begin{bmatrix} 1 \\ 3 \end{bmatrix} + x_2 \begin{bmatrix} -3 \\ -7 \end{bmatrix} + x_3 \begin{bmatrix} 2 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 8 \\ 0 \end{bmatrix}$

M.E. $\quad \begin{bmatrix} 1 & -3 & 2 & -1 \\ 3 & -7 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 8 \\ 0 \end{bmatrix}$

Augmented matrix for solving L.S.

$\begin{bmatrix} 1 & -3 & 2 & -1 & | & 8 \\ 3 & -7 & 0 & 1 & | & 0 \end{bmatrix}$ $-3R_1 + R_2 \rightarrow R_2$ $\begin{bmatrix} 1 & -3 & 2 & -1 & | & 8 \\ 0 & 2 & -6 & 4 & | & -24 \end{bmatrix}$

$\frac{1}{2}R_2 \rightarrow R_2$ $\begin{bmatrix} 1 & -3 & 2 & -1 & | & 8 \\ 0 & 1 & -3 & 2 & | & -12 \end{bmatrix}$

$3R_2 + R_1 \rightarrow R_1$ $\begin{bmatrix} 1 & 0 & -7 & 5 & | & -28 \\ 0 & 1 & -3 & 2 & | & -12 \end{bmatrix}$

$\begin{cases} x_1 - 7x_3 + 5x_4 = -28 \\ x_2 - 3x_3 + 2x_4 = -12 \\ x_1, x_2 \text{ Basic} \\ x_3, x_4 \text{ free} \end{cases}$

$X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -28 + 7x_3 - 5x_4 \\ -12 + 3x_3 - 2x_4 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -28 \\ -12 \\ 0 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} 7 \\ 3 \\ 1 \\ 0 \end{pmatrix} + x_4 \begin{pmatrix} -5 \\ -2 \\ 0 \\ 1 \end{pmatrix}$