

$$A = \begin{bmatrix} 3 & 2 \\ 3 & -2 \end{bmatrix}$$

$$\begin{aligned} p(\lambda) &= \det(A - \lambda I) = \det \begin{bmatrix} 3-\lambda & 2 \\ 3 & -2-\lambda \end{bmatrix} \\ &= (3-\lambda)(-2-\lambda) - 6 \\ &= \lambda^2 - \lambda - 12 \end{aligned}$$

$$\begin{aligned} \lambda^2 - \lambda - 12 = 0 &\Rightarrow (\lambda - 4)(\lambda + 3) = 0 \\ &\lambda = 4 \text{ or } \lambda = -3 \end{aligned}$$

$$\lambda = 4$$

$$(A - 4I)x = 0$$

$$\begin{bmatrix} -1 & 2 \\ 3 & -6 \end{bmatrix} \vec{x} = \vec{0}$$

$$\left[\begin{array}{cc|c} -1 & 2 & 0 \\ 3 & -6 & 0 \end{array} \right] \sim \left[\begin{array}{cc|c} 1 & -2 & 0 \\ 0 & 0 & 0 \end{array} \right]$$

$$\begin{aligned} x_1 &= 2x_2 \\ x_2 &= x_2 \Rightarrow x = x_2 \begin{pmatrix} 2 \\ 1 \end{pmatrix} \end{aligned}$$

$$\lambda = 4 \quad x = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$\lambda = -3 \quad (A + 3I)x = 0$$

$$\left[\begin{array}{cc|c} 6 & 2 \\ 3 & 1 \end{array} \right] \vec{x} = \vec{0} \Rightarrow \left[\begin{array}{cc|c} 6 & 2 & 0 \\ 3 & 1 & 0 \end{array} \right] \sim \left[\begin{array}{cc|c} 1 & \frac{1}{3} & 0 \\ 0 & 0 & 0 \end{array} \right]$$

$$\begin{aligned} x_1 &= -\frac{1}{3}x_2 \\ x_2 &= x_2 \Rightarrow x = x_2 \begin{pmatrix} -\frac{1}{3} \\ 1 \end{pmatrix} \text{ or } x_2 \begin{pmatrix} -1 \\ 3 \end{pmatrix} \end{aligned}$$

$$\lambda = -3 \quad x = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$$

$$B = \begin{bmatrix} -1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 3 & -1 \end{bmatrix}$$

$$p(\lambda) = \det(B - \lambda I) = \det \begin{bmatrix} -1-\lambda & 1 & 0 \\ 1 & 2-\lambda & 1 \\ 0 & 3 & -1-\lambda \end{bmatrix} = 0$$

$$\Rightarrow (-1-\lambda)(\lambda-3)(\lambda+2) = 0$$

$$\lambda = -1, \lambda = 3, \text{ or } \lambda = -2$$

$$\lambda = -1 \quad (B + I)x = 0 \Rightarrow \begin{bmatrix} 0 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 3 & 0 \end{bmatrix} x = 0$$

$$\left[\begin{array}{ccc|c} 0 & 1 & 0 & 0 \\ 1 & 3 & 1 & 0 \\ 0 & 3 & 0 & 0 \end{array} \right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \Rightarrow \begin{aligned} x_1 + x_3 &= 0 \\ x_2 &= 0 \end{aligned}$$

$$\lambda = -1 \quad x = x_3 \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \Rightarrow \lambda = -1 \quad x = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

$$\lambda = 3 \quad (B - 3I)x = 0 \Rightarrow \begin{bmatrix} -4 & 1 & 0 \\ 1 & -1 & 1 \\ 0 & 3 & -4 \end{bmatrix} \vec{x} = 0$$

$$\left[\begin{array}{ccc|c} -4 & 1 & 0 & 0 \\ 1 & -1 & 1 & 0 \\ 0 & 3 & -4 & 0 \end{array} \right] \sim \left[\begin{array}{ccc|c} 1 & 0 & -\frac{1}{3} & 0 \\ 0 & 1 & -\frac{4}{3} & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \Rightarrow \begin{aligned} x_1 - \frac{1}{3}x_3 &= 0 \\ x_2 - \frac{4}{3}x_3 &= 0 \\ x_3 &= x_3 \end{aligned}$$

$$x = x_3 \begin{pmatrix} \frac{1}{3} \\ \frac{4}{3} \\ 1 \end{pmatrix} = x_3 \begin{pmatrix} 1 \\ 4 \\ 3 \end{pmatrix} \Rightarrow \lambda = 3 \quad x = \begin{pmatrix} 1 \\ 4 \\ 3 \end{pmatrix}$$

$$\lambda = -2 \quad (B + 2I)x = 0 \Rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 3 & 1 \end{bmatrix} x = 0$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 1 & 4 & 1 & 0 \\ 0 & 3 & 1 & 0 \end{array} \right] \sim \left[\begin{array}{ccc|c} 1 & 0 & -\frac{1}{3} & 0 \\ 0 & 1 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \Rightarrow \begin{aligned} x_1 - \frac{1}{3}x_3 &= 0 \\ x_2 + \frac{1}{3}x_3 &= 0 \\ x_3 &= x_3 \end{aligned}$$

$$x = x_3 \begin{pmatrix} \frac{1}{3} \\ -\frac{1}{3} \\ 1 \end{pmatrix} = x_3 \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$$

$$\Rightarrow \lambda = -2 \quad x = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$$