# ANSWERS

**MATH 215**
**Practice**

Solve the given systems of equations.

$$\begin{aligned} x_1 + 2x_2 - 3x_3 &= -1 \\ 5x_1 + 10x_2 - 7x_3 &= -2 \\ 3x_1 + 6x_2 + 5x_3 &= 11 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & -1 \\ 5 & 10 & -7 & -2 \\ 3 & 6 & 5 & 11 \end{array}\right]$$

$-5R_1 + R_2 \to R_2$

$-3R_1 + R_3 \to R_3$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & -1 \\ 0 & 0 & 8 & 3 \\ 0 & 0 & 14 & 14 \end{array}\right]$$

$\frac{1}{14}R_2 \to R_2$

$R_2 \leftrightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 8 & 3 \end{array}\right]$$

$-8R_2 + R_3 \to R_3$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & -5 \end{array}\right]$$

$0 \neq -5$ No Solution

$$\begin{aligned} 3x_1 - x_2 - 5x_3 &= 9 \\ x_2 - 10x_3 &= 0 \\ 2x_1 - x_2 &= 6 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 3 & -1 & -5 & 9 \\ 0 & 1 & -10 & 0 \\ 2 & -1 & 0 & 6 \end{array}\right]$$

$3R_3 \to R_3$

$$\left[\begin{array}{ccc|c} 3 & -1 & -5 & 9 \\ 0 & 1 & -10 & 0 \\ 6 & -3 & 0 & 18 \end{array}\right]$$

$-2R_1 + R_3 \to R_3$

$$\left[\begin{array}{ccc|c} 3 & -1 & -5 & 9 \\ 0 & 1 & -10 & 0 \\ 0 & -1 & 10 & 0 \end{array}\right]$$

$R_2 + R_1 \to R_1$

$R_2 + R_3 \to R_3$

$$\left[\begin{array}{ccc|c} 3 & 0 & -15 & 9 \\ 0 & 1 & -10 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\frac{1}{3}R_1 \to R_1$

$$\left[\begin{array}{ccc|c} 1 & 0 & -5 & 3 \\ 0 & 1 & -10 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$ R.R.E.F.

$x_1 - 5x_3 = 3$

$x_2 - 10x_3 = 0$

$x_1 = 3 + 5x_3$

$x_2 = 10x_3$

$x_3 = x_3$

$x_1, x_2$ Basic

$x_3$ free

Inf # of Solutions

$$\begin{aligned} x_1 + 2x_2 + x_3 &= 3 \\ 3x_1 - x_2 - 3x_3 &= -1 \\ 2x_1 + 3x_2 + x_3 &= 4 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 3 & -1 & -3 & -1 \\ 2 & 3 & 1 & 4 \end{array}\right]$$

$-3R_1 + R_2 \to R_2$

$-2R_1 + R_3 \to R_3$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & -7 & -6 & -10 \\ 0 & -1 & -1 & -2 \end{array}\right]$$

$-R_2 \to R_2$

$R_2 \leftrightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & 1 & 1 & 2 \\ 0 & -7 & -6 & -10 \end{array}\right]$$

$-2R_2 + R_1 \to R_1$

$7R_2 + R_3 \to R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & -1 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

$-R_3 + R_2 \to R_2$

$R_3 + R_1 \to R_1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 4 \end{array}\right]$$ R.R.E.F.

$x_1 = 3$

$x_2 = -2$

$x_3 = 4$

Unique Solution