Chapter 21: Saving Models

For All Practical Purposes

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Section 21.3 A Limit to Compounding

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A Limit to Compound Interest

- The following table shows a trend: More frequent compounding yields more interest.
- As the frequency of compounding increases, the interest tends to reach a limiting amount (shown in the right columns).

Comparing Compound Interest The Value of $1000 at 10% Annual Interest, for Different Compounding Periods					
Years	Compounded Yearly	Compounded Quarterly	Compounded Monthly	Compounded Daily	Compounded Continuously
1	1100.00	1103.81	1104.71	1105.16	1105.17
5	1610.51	1638.62	1645.31	1648.61	1648.72
10	2593.74	2685.06	2707.04	2717.91	2718.28

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Continuous Compounding

- As n gets very large, $(1 + {}^{1}/_{n})^{n}$ approaches the constant $e \approx 2.71828$.
- For a principal P deposited in an account at a nominal annual rate r, compounded continuously, the balance after t years is:

$$A = P\, e^{r t}$$

Example: For $1000 at an annual rate of 10%, compounded n times in the course of a single year, what is the balance at the end of the year?

As the quantity gets closer and closer to $1000($e^{0.1}$) = $1105.17.

No matter how frequently interest is compounded, the original $1000 at the end of one year cannot grow beyond $1105.17.

Yield of $1 at 100% Interest ($i$ = 1) Compounded n Times per Year	
n	$(1 + {}^{1}/_{n})^{n}$
1	2.0000000
5	2.4883200
10	2.5937424
50	2.6915880
100	2.7048138
1,000	2.7169239
10,000	2.7181459
100,000	2.7182682
1,000,000	2.7182818
10,000,000	2.7182818

It approaches $e \approx 2.71828$ (which is the base of the natural logarithms).