Chapter 1: Urban Services

Section 1.3 Beyond Euler Circuits

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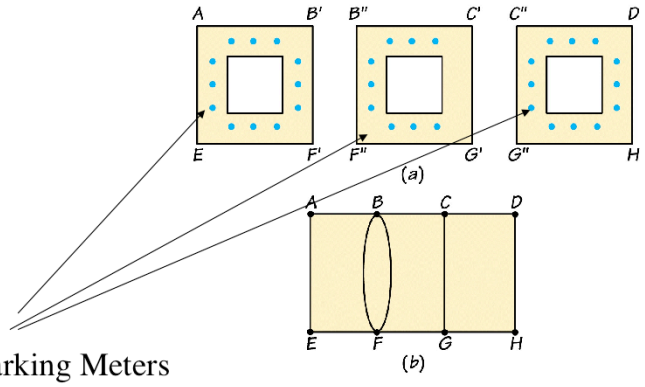
In real life, not all problems will be perfect Euler circuits. If no Euler circuit exists (odd valences), you want to minimize the length of the circuit by carefully choosing the edges to be retraced. For our purposes, we assume all edges have the same length—*simplified* Chinese postman problem. Named after Meigu Guan

Chinese postman problem – Minimize the length of the circuit by carefully choosing which edges to retrace. (All edges have the same length.)



No Euler Circuit
Why ?

Parking Meters

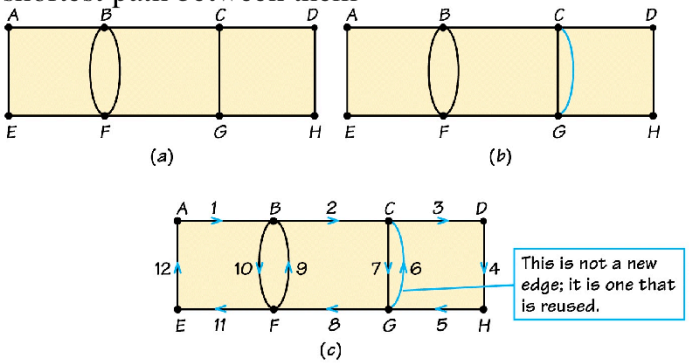
Eulerizing the graph by adding edges to a graph.

1.) Add edges until the graph is connected and has even valence. (New graph has an Euler circuit.)

2.) Find an Euler circuit

3.) Typically the number of reuses of edges equals the number of edges added during eulerization.

4.) If two vertices have odd valence, add edges along the shortest path between them



Using the Eulerizing graph applet in the companion website is very helpful here!!!

Eulerizing a Graph

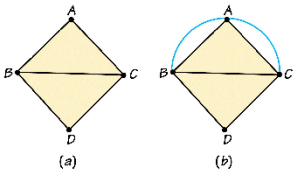
1. On the graph, add edges by duplicating existing ones, until you arrive at a graph that is connected and even-valent.

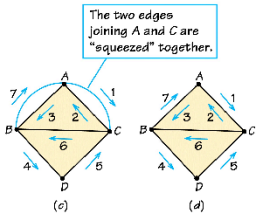
   There are many ways to Eulerize a graph. The graph below is an efficient eulerization because the fewest number of edges were added.

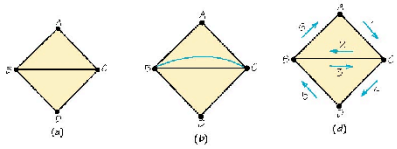
2. Find an Euler circuit on the eulerized graph.

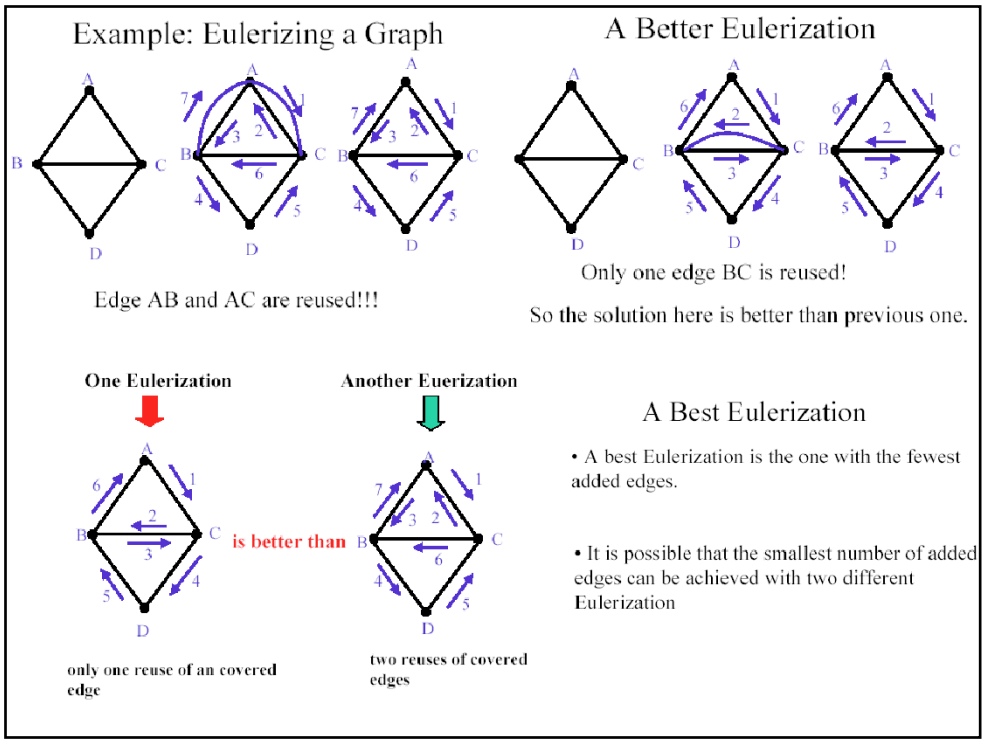
   Traverse every original and "added" edge once, as you find a circuit that starts and ends at the same vertex.

3. "Squeeze" this Euler circuit from the eulerized graph onto the original graph by replacing the "added" edge with an arrow showing it was retraced.



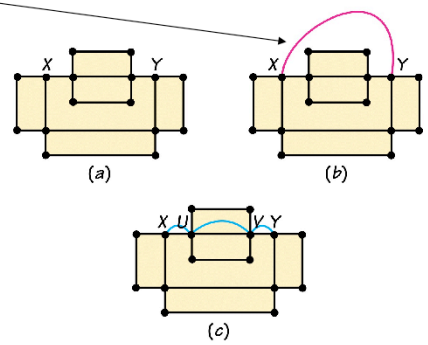






Eulerizing a graph is not unique and every Eulerization of the graph may not be the best.

Street does not exist



- Hints for Eulerizing a Graph
  - For the most efficient eulerization, look for the fewest edges to add to make all vertices even.
  - Typically, locate odd valence vertices and try to reuse (add) the connecting edge between the vertices.
  - Sometimes vertices are more than one edge apart; in this case, reuse edges between vertices (see graph below).

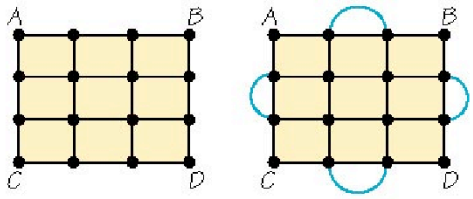
Remember: Only duplicate (add to) the existing edges.

Cannot add a street that does not exist. Think about driving on the roads.

- Rectangular Networks – This is the name given to a street network composed of a series of rectangular blocks that form a large rectangle made up of so many blocks high by so many blocks wide.

Eulerizing rectangular networks:
"Edge Walker"

- Start in upper left corner (at A).
- Travel (clockwise) around the outer boundary.
- As you travel, add an edge by the following rules:



1. If the vertex is odd, add an edge by linking it to the next vertex.
   - If this next vertex becomes even, skip it (just keep "walking").
   - If this next vertex becomes odd, (on a corner) link it to the next vertex.
2. Repeat this rule until you reach the upper left corner again.

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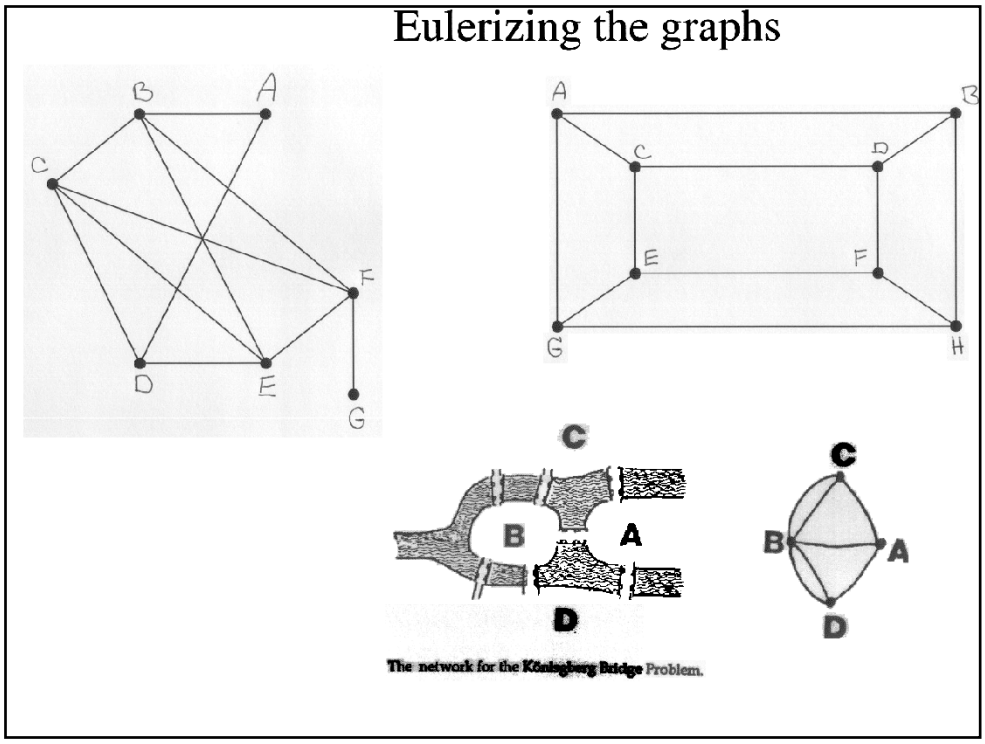

Try Eulerizing these graphs. However, for the online class, it is easier to use the Eulerizing Graph applet in the companion website for practice.