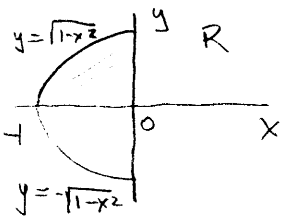
Quiz 6 – Solutions

#16, p.759

The region of integration R is:



In polar coordinates:

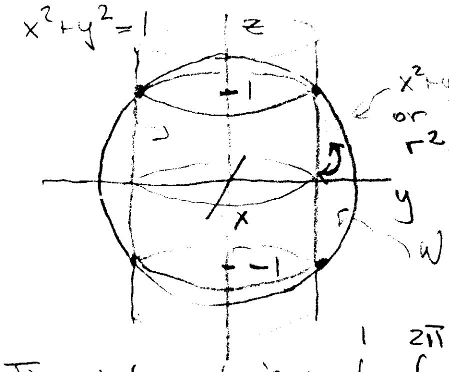
$R: \ 0 \le r \le 1, \ \frac{\pi}{2} \le \theta \le \frac{3\pi}{2}$. Hence,

$$\int_{-1}^{0}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} x\,dy\,dx = \int_R x\,dA = \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}}\int_0^1 r\cos\theta\, r\,dr\,d\theta =$$

$$= \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \frac{1}{3}\cos\theta\, d\theta = -\frac{2}{3}.$$

($x = r\cos\theta$, $dA = r\,dr\,d\theta$ in polar coordinates.

#14, p.766 Set up the integral in cylindrical coordinates.



When $x^2+y^2=1$, $x^2+y^2+z^2=1+z^2$.
So points in the intersection of the sphere and the cylinder satisfy:

$1+z^2=2$, $z=\pm 1$.

Hence, W is the region pictured to the left and in cylindrical coordinates:

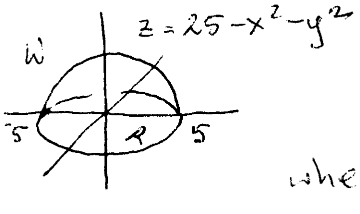
$W: \ 0 \le \theta \le 2\pi, \ -1 \le z \le 1, \ 1 \le r \le \sqrt{2-z^2}$

The integral is: $\int_{-1}^{1}\int_0^{2\pi}\int_1^{\sqrt{2-z^2}} r^2\, r\,dr\,d\theta\,dz = \int_{-1}^{1}\int_0^{2\pi} \frac{1}{4}r^4\Big|_1^{\sqrt{2-z^2}} d\theta\,dz =$

$$= \frac{1}{4}\int_{-1}^{1}\int_0^{2\pi}((2-z)^2-1)\,d\theta\,dz = \frac{28\pi}{15}.$$

#20, 760

$z = 25-x^2-y^2$ is an upside down paraboloid. It intersects the plane $z=0$ along the circle $x^2+y^2=25$. Hence, the volume we are looking for is:



$$\int_R (25-x^2-y^2)\,dA = \int_0^{2\pi}\int_0^{5}(25-r^2)\, r\,dr\,d\theta = \frac{625\pi}{2},$$

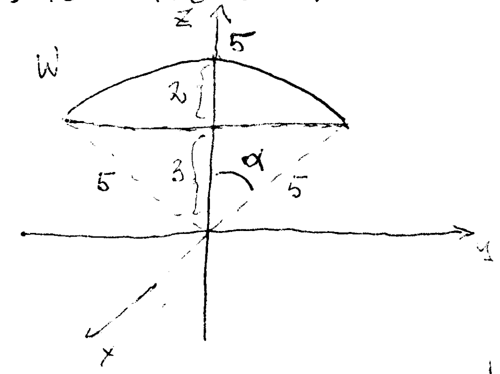
where R is the circle of radius 5 on the xy-plane.

The same volume can be found using a triple integral. Let W be the solid under the paraboloid. Then, in cylindrical coordinates:

$$\mathrm{vol}(W) = \int_W 1\,dV = \int_0^{2\pi}\int_0^{5}\int_0^{25-r^2} 1\, r\,dz\,dr\,d\theta = \int_0^{2\pi}\int_0^{5}(25-r^2)\, r\,dr\,d\theta = \frac{625\pi}{2}$$

#24, p. 767

Orient the solid shown as follows:



Use the spherical coordinates with the origin at the center of the sphere. To set up an iterated integral

$$\int_W 1\,dV = \mathrm{vol}(W)$$

in spherical coordinates we need the value of the angle α at the vertex. Given α, W in spherical coordinates is:

$$W: \quad 0 \le \theta \le 2\pi \;, \quad 0 \le \Phi \le \alpha \;, \quad \frac{3}{\cos\Phi} \le \rho \le 5 .$$

Indeed, for each θ and Φ, the lower limit for ρ is on the plane $z=3$ which in spherical coordinates has the equation $\rho = \dfrac{3}{\cos\Phi}$.

To find α, observe that $\alpha = \arccos\left(\frac{3}{5}\right)$. Thus:

$$\int_W 1\,dV = \int_0^{\arccos(\frac{3}{5})} \int_0^{2\pi} \int_{\frac{3}{\cos\Phi}}^{5} \rho^2 \sin\Phi \, d\rho \, d\theta \, d\Phi =$$

$$= \int_0^{\arccos(\frac{3}{5})} \int_0^{2\pi} \left(\frac{125}{3} - \frac{9}{\cos^3\Phi} \right) \sin\Phi \, d\theta \, d\Phi =$$

$$= \int_0^{\arccos(\frac{3}{5})} 2\pi \left(\frac{125}{3} \sin\Phi - \frac{9\sin\Phi}{\cos^3\Phi} \right) d\Phi =$$

$$= \frac{52\pi}{3} \cong 54.45 \text{ cm}^3$$

(For the last integral, you could have used your calculator or find an antiderivative.)