Name: _____

MTH 243 Sec 2 *Quiz 5* *March 24, 2005*

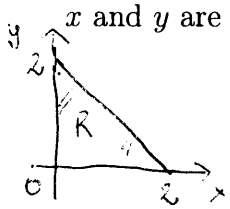
1. Find the triple integral of the function $h(x,y,z) = x + 2y + z$ over the solid W described by: $0 \le x \le 2, 0 \le y \le 1, 0 \le z \le 1$.

$$\iiint_W h(x,y,z)\,dV = \int_0^2 \int_0^1 \int_0^1 (x+2y+z)\,dy\,dz\,dx = \int_0^2 \int_0^1 \left((xy + y^2 + zy)\Big|_0^1 \right) dz\,dx =$$

$$= \int_0^2 \int_0^1 (x + 1 + z)\,dz\,dx = \int_0^2 \left((xz + z + \tfrac{1}{2}z^2)\Big|_0^1 \right) dx =$$

$$= \int_0^2 (x + \tfrac{3}{2})\,dx = \underline{5}$$

2. Find the total mass M of a metal plate that occupies the region in the xy-plane bounded by $x = 0$, $y = 0$, and $y = 2 - x$. The density of mass is given by $\delta(x,y) = 2 + xy$ grams per cm^2, x and y are measured in centimeters.

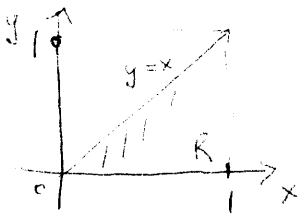


$$M = \iint_R \delta(x,y)\,dA = \int_0^2 \int_0^{2-x} (2 + xy)\,dy\,dx =$$

$$= \int_0^2 \left(\left(2y + x\frac{y^2}{2} \right)\Big|_0^{2-x} \right) dx =$$

$$= \int_0^2 \left(2(2-x) + x(2-x)^2 \cdot \tfrac{1}{2} \right) dx = \frac{14}{3} \cong 4.66 .$$

3. For the integral $\int_0^1 \int_y^1 \sin(x^2)\,dx\,dy$, sketch the region of integration. Reverse the order of integration and evaluate the integral.



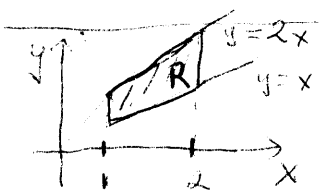
$$\iint_R \sin(x^2)\,dA = \int_0^1 \int_y^1 \sin(x^2)\,dx\,dy =$$

$$= \int_0^1 \int_0^x \sin(x^2)\,dy\,dx = \int_0^1 x \sin(x^2)\,dx =$$

$$= -\tfrac{1}{2}\cos(x^2)\Big|_0^1 = -\tfrac{1}{2}\cos(1) + \tfrac{1}{2} \cong 0.229$$

4. For the integral

$$\int_1^2 \int_x^{2x} (x^2 y)\,dy\,dx$$



Sketch the region of integration and evaluate the integral.

$$\int_1^2 \int_x^{2x} (x^2 y)\,dy\,dx = \int_1^2 \left(\frac{x^2 y^2}{2}\Big|_x^{2x} \right) dx = \int_1^2 \left(\frac{4x^4}{2} - \frac{x^4}{2} \right) dx = \int_1^2 \frac{3x^4}{2}\,dx \cong \underline{9.3}$$

Name: _____

MTH 243 Sec 3 *Quiz 5* *March 24, 2005*

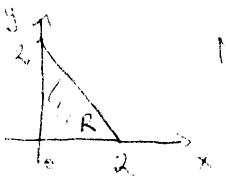
1. Find the triple integral of the function $h(x,y,z) = 2x+y+z$ over the solid W described by: $0 \le x \le 1, 0 \le y \le 2, 0 \le z \le 1$.

$$\iiint_W h(x,y,z)\,dV = \int_0^2\int_0^1\int_0^1 (2x+y+z)\,dx\,dz\,dy = \int_0^2\int_0^1 \left((x^2+xy+xz)\Big|_0^1\right)dz\,dy =$$

$$= \int_0^2\int_0^1 (1+y+z)\,dz\,dy = \int_0^2 \left((z+yz+\tfrac{1}{2}z^2)\Big|_0^1\right)dy =$$

$$= \int_0^2 (y+\tfrac{3}{2})\,dy = \underline{5}$$

2. Find the total mass M of a metal plate that occupies the region in the xy-plane bounded by $x = 0$, $y = 0$, and $y = 2 - x$. The density of mass is given by $\delta(x,y) = 1 + 2xy$ grams per cm^2, x and y are measured in centimeters.

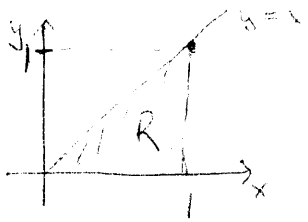


$$M = \iint_R (1+2xy)\,dA = \int_0^2\int_0^{2-x} (1+2xy)\,dy\,dx =$$

$$\int_0^2 \left((y+xy^2)\Big|_0^{2-x}\right)dx = \int_0^2 \left((2-x)+x(2-x)^2\right)dx = \frac{10}{3}$$

$$\approx 3.33$$

3. For the integral $\int_0^1\int_y^1 \cos(x^2)\,dx\,dy$, sketch the region of integration. Reverse the order of integration and evaluate the integral.

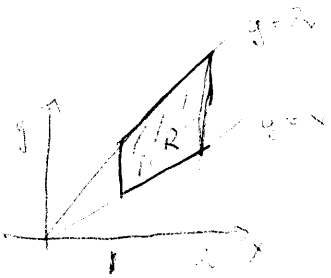


$$\int_0^1\int_y^1 \cos(x^2)\,dx\,dy = \int_0^1\int_0^x \cos(x^2)\,dy\,dx =$$

$$= \int_0^1 x\cos(x^2)\,dx = \tfrac{1}{2}\sin(x^2)\Big|_0^1 = \tfrac{1}{2}\sin(1) \approx 0.421$$

4. For the integral

$$\int_1^2\int_x^{2x} (2x^2y)\,dy\,dx$$



Sketch the region of integration and evaluate the integral.

$$\int_1^2\int_x^{2x} (2x^2y)\,dy\,dx = \int_1^2 \left(x^2y^2\Big|_x^{2x}\right)dx = \int_1^2 3x^4\,dx = \frac{93}{5} \approx 18.6$$