Name: ______________________________

MTH 243 Sec 2 *Quiz 4* *March 10, 2005*

Explain all answers. Show all steps.

1. (a) Let $z = f(x, y)$, $x = g(t, s)$, $y = h(t, s)$. Write the formula for $\frac{\partial z}{\partial t}$.

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial t}.$$

(b) Let $z = x^2y + 3y^2$. Let $x = g(t)$, $y = h(t)$. Assume that $g(2) = -1$, $h(2) = 3$, $g'(2) = 0.3$, $h'(2) = -1.2$. Find $\frac{\partial z}{\partial t}\big|_{t=2}$.

At $t = 2$, $x = -1$, $y = 3$. Hence:

$$\frac{\partial z}{\partial t}\Big|_{t=2} = \frac{\partial z}{\partial x}\Big|_{(x,y)=(-1,3)} \cdot \frac{dx}{dt}\Big|_{t=2} + \frac{\partial z}{\partial y}\Big|_{(x,y)=(-1,3)} \cdot \frac{dy}{dt}\Big|_{t=2} =$$

$$= (2xy)\big|_{(-1,3)} \cdot 0.3 + (x^2 + 6y)\big|_{(-1,3)} \cdot (-1.2) = -24.6$$

$=$

3. Let $f(x, y) = x^3 + y^3 - 6y^2 - 3x$. Find all critical points of f and classify them as local minima, local maxima or saddle points.

See Problem 3 for Sec 3. Everything is the same as the constant 9 does not affect the derivatives of $f(x,y)$.

Name: ______________________________

MTH 243 Sec 3 *Quiz 4* *March 10, 2005*

Explain all answers. Show all steps.

1. (a) Let $z = f(x, y)$, $x = g(t, s)$, $y = h(t, s)$. Write the formula for $\frac{\partial z}{\partial s}$.

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial s}$$

(b) Let $z = x^2y + 3y^2$. Let $x = g(t)$, $y = h(t)$. Assume that $g(1) = 2$, $h(1) = 3$, $g'(1) = -0.3$, $h'(1) = 1.2$. Find $\frac{\partial z}{\partial t}\big|_{t=1}$.

At $t=1$, $x = 2$; $y = 3$. Hence

$$\frac{\partial z}{\partial t}\Big|_{t=1} = \frac{\partial z}{\partial x}\Big|_{(x,y)=(2,3)} \cdot \frac{dx}{dt}\Big|_{t=1} + \frac{\partial z}{\partial y}\Big|_{(x,y)=(2,3)} \cdot \frac{dy}{dt}\Big|_{t=1} =$$

$$= (2xy)\big|_{(2,3)} \cdot (-0.3) + (x^2 + 6y)\big|_{(2,3)} \cdot 1.2 = \underline{22.8}$$

3. Let $f(x, y) = x^3 + y^3 - 6y^2 - 3x + 9$. Find all critical points of f and classify them as local minima, local maxima or saddle points.

$f_x = 3x^2 - 3$, $f_y = 3y^2 - 12y$ Critical points:

$$\begin{cases} 3x^2 - 3 = 0 \\ 3y^2 - 12y = 0 \end{cases} \rightarrow (-1,0), (1,0), (-1,4), (1,4)$$

$f_{xx} = 6x$, $f_{yy} = 6y - 12$, $f_{xy} = 0$. Hence,

$D(x,y) = 6x(6y - 12)$

At $(-1,0)$:

$D(-1,0) > 0$, $f_{xx}(-1,0) < 0$ $\rightarrow$ $(-1,0)$ is a local maximum

At $(1,0)$:

$D(1,0) < 0$ $\rightarrow$ $(1,0)$ is a saddle point

At $(1,4)$:

$D(1,4) > 0$, $f_{xx}(1,4) > 0$ $\rightarrow$ $(1,4)$ is a local minimum

At $(-1,4)$:

$D(-1,4) < 0$ $\rightarrow$ $(-1,4)$ is a saddle point.