16.3 # 6

6. The region is the half cylinder in Figure 16.42.

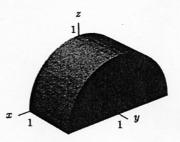


Figure 16.42

163 # 16

(a) The equation of the surface of the whole cylinder along the y-axis is $x^2 + z^2 = 1$. The part we want is $z = \sqrt{1 - x^2}$ $0 \le y \le 10$.

See Figure 16.52.



Figure 16.52

(b) The integral is

$$\int_{D} f(x,y,z) \, dV = \int_{0}^{10} \int_{-1}^{1} \int_{0}^{\sqrt{1-x^2}} f(x,y,z) \, dz dx dy.$$

16.4 # 14

The presence of the term x^2+y^2 suggests that we should convert the integral into polar coordinates. Since $\sqrt{x^2+y^2}=r$, the integral becomes

$$\int_{R} \sqrt{x^2 + y^2} \, dx dy = \int_{0}^{2\pi} \int_{2}^{3} r^2 \, dr d\theta = \int_{0}^{2\pi} \left. \frac{r^3}{3} \right|_{2}^{3} \, d\theta = \int_{0}^{2\pi} \frac{19}{3} \, d\theta = \frac{38\pi}{3}.$$

By the given limits
$$0 \le x \le -1$$
, and $-\sqrt{1-x^2} \le y \le \sqrt{1-x^2}$, the region of integration is in Figure 16.66.

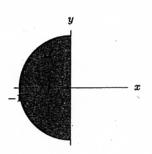


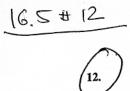
Figure 16.66

In polar coordinates, we have

$$\int_{\pi/2}^{3\pi/2} \int_{0}^{1} r \cos \theta \, r \, dr \, d\theta = \int_{\pi/2}^{3\pi/2} \cos \theta \left(\frac{1}{3}r^{3}\right) \Big|_{0}^{1} d\theta$$
$$= \frac{1}{3} \int_{\pi/2}^{3\pi/2} \cos \theta \, d\theta$$
$$= \frac{1}{3} \sin \theta \Big|_{\pi/2}^{3\pi/2} = \frac{1}{3} (-1 - 1) = -\frac{2}{3}$$

$$\int_0^1 \int_0^{2\pi} \int_0^4 f \cdot r dr d\theta dz$$

$$\int_0^\pi \int_0^\pi \int_2^3 f \cdot \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta$$



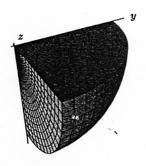


Figure 16.75

R is one eighth of a sphere of radius 1, below the xy-plane and under the first quadrant.