MTH 111 Exam 1 February 20, 2002

YOUR NAME: **YOUR INSTRUCTOR'S NAME:**

Part I -- NO CALCULATORS ALLOWED. You are not allowed to have your calculator on your desk while working on Part I. After you finish, hand in Part I. Then you can use your calculator.

The problems below seem like multiple-choice problems, but indeed they are not. You have to show your work, no credit will be given without work. Partial credit is possible. Part I consists of 8 problems worth 5 points each.

1. Simplify

$$(x^3 y^4)^2 (3xy^2)^3 .$$

(A) $27xy^2$ (B) $27x^9y^{14}$ (C) $27x^8y^{11}$ (D) $9x^8y^{11}$

Show your work:

$$(x^3y^4)^2(3xy^2)^3 = x^6y^8 \cdot 27x^3y^6 = 27x^9y^{14}$$

2. Simplify

$$\frac{a^{(-3)}b^2}{a^3b^{(-2)}} .$$

(A) 1 (B) $\frac{a^6}{b^4}$ (C) $\frac{b^4}{a^6}$ (D) $\frac{b^4}{a^9}$

Show your work:

$$\frac{a^{-3}b^2}{a^3b^{-2}} = \frac{a^{-3}b^2 \cdot a^3b^2}{a^3b^{-2} \cdot a^3b^2} = \frac{b^4}{a^6}$$

3. Simplify

$$\frac{a+b}{a^2+b^2}.$$

(A) $a+b$ **(B)** $\frac{1}{a+b}$ **(C)** $\frac{1}{2(a+b)}$ **(D)** None of the above

Show your work:

$\frac{a+b}{a^2+b^2}$ can't be simplified

4. Simplify

$$\frac{x^2-3x}{x^2-9}.$$

(A) $\frac{1-x}{-2}$ **(B)** $\frac{x}{3}$ **(C)** $\frac{x}{x+3}$ **(D)** None of the above

Show your work:

$$\frac{x^2-3x}{x^2-9} = \frac{x(x-3)}{(x-3)(x+3)} = \frac{x}{x+3}$$

5. Simplify as far as possible

$$\frac{x^2-4}{x^2-4x+4}.$$

(A) $\frac{1}{x+4}$ **(B)** $\frac{x+2}{x-2}$ **(C)** $\frac{1}{x+1}$ **(D)** None of the above

Show your work:

$$\frac{x^2-4}{x^2-4x+4} = \frac{(x-2)(x+2)}{(x-2)^2} = \frac{x+2}{x-2}$$

6. Find the lowest common denominator, write as one quotient, simplify

$$\frac{2}{x^2-4}-\frac{3}{x-2}.$$

(A) $\frac{-x^2+7x+9}{(x^2-4)(x-2)}$ (B) $\frac{-3x-4}{x^2-4}$ (C) $\frac{-3x+8}{x^2-4}$ (D) $\frac{-1}{x-1}$

Show your work:

$$\frac{2}{x^2-4}-\frac{3}{x-2}=\frac{2}{(x-2)(x+2)}-\frac{3(x+2)}{(x-2)(x+2)}=$$

$$=\frac{2}{x^2-4}-\frac{3x+6}{x^2-4}=\frac{2-3x-6}{x^2-4}=\frac{-3x-4}{x^2-4}$$

7. Solve for x

$$3(5-2x)=10-8(x-1).$$

Show your work:

$$15-6x=10-8x+8$$

$$2x=3$$

$$x=\frac{3}{2}$$

8. Simplify

$$\frac{c-\frac{4}{c^2}}{1+\frac{1}{c}}.$$

(A) $\frac{c-4}{c}$ (B) $\frac{c^3-4}{c+1}$ (C) $\frac{c^3-4}{c^2+c}$ (D) $\frac{c-4}{1+c}$

Show your work:

$$\frac{c-\frac{4}{c^2}}{1+\frac{1}{c}}=\frac{c^2\left(c-\frac{4}{c^2}\right)}{c^2\left(1+\frac{1}{c}\right)}=\frac{c^3-4}{c^2+c}$$

Part II -- YOU CAN USE YOUR CALCULATORS IN THIS PART.

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Part II consists of 10 problems each worth 6 points. Be sure to explain all your answers! No credit without explanations.

9. For $f(x) = 2 - 3x^2$ find:

(a) $f(5)$ **(b)** $f(-5)$ **(c)** $f(1+h) - f(1)$.

In (c) simplify your answer as far as possible.

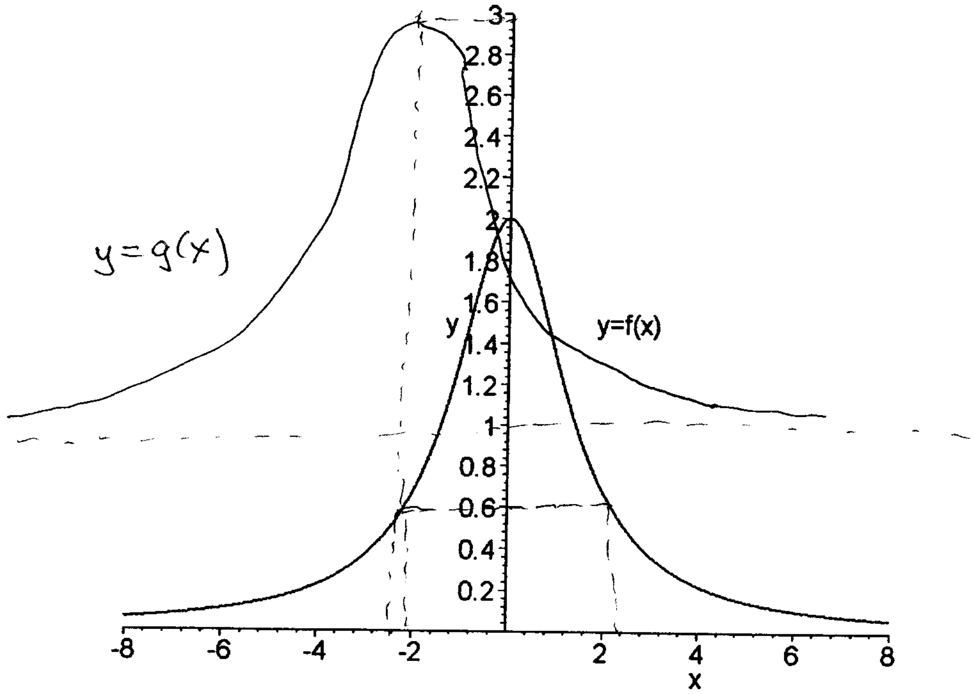
$f(5) = 2 - 3 \cdot 5^2 = \underline{-73}$, $f(-5) = 2 - 3(-5)^2 = \underline{-73}$

$f(1+h) - f(1) = 2 - 3(1+h)^2 - (2-3) =$

$= 2 - 3(1 + 2h + h^2) - (-1) = 2 - 3 - 6h - 3h^2 + 1 =$

$= \underline{-6h - 3h^2}$

Below you see the graph of a function $f(x) = \dfrac{4}{x^2+2}$ on the interval (-8,8). Answer the following questions **10-13** regarding the function $y = f(x)$.



10. Write a formula for the function $g(x)$ whose graph is obtained from the graph $f(x)$ by shifting vertically 1 unit up, and horizontally 2 units to the left. Draw the graph of $g(x)$ in the same coordinate system above.

$$g(x) = f(x+2) + 1$$

$$g(x) = \frac{4}{(x+2)^2+2} + 1$$

11. Determine intervals (within (-8,8)) on which the function $y = f(x)$ given above is

(a) increasing: $(-8, 0)$

(b) decreasing: $(0, 8)$

(c) constant: —

(d) $f(x)$ has a relative maximum at $x =$ 0

12. Based only on the graph of the function $y = f(x)$ above, answer the following question: how many values x are there for which $f(x) = 0.6$? Estimate those values from the graph. Explain your reasoning.

Two values of x, approximately $x \cong 2.4$, $x \cong -2.4$

13. The function $y = f(x)$ in the graph above is

(a) even **(b)** odd **(c)** neither.

(Explain your answer both geometrically, using the graph, and algebraically using the formula for $f(x)$.)

$f(x)$ is even. Geometrically, it is evident on the graph $y = f(x)$ is symmetric with respect to the y-axis.

Algebraically: $f(-x) = \frac{4}{(-x)^2+2} = \frac{4}{x^2+2} = f(x)$.

14. Find an equation of the line that passes through (1,-2) and is perpendicular to the line $5y + x = 15$.

$5y + x = 15$ is equivalent to $y = -\frac{1}{5}x + 3$.

The slope m of any perpendicular line is

$$m = -\frac{1}{-\frac{1}{5}} = 5$$

Hence, the equation of our line is $5(x-1) = y+2$, or

$y = 5x - 7$

15. Use your calculator to calculate the value of

$$\left(\frac{4.1^2+2}{(1.2-1)^3+20}\right)^2.$$

Give at least four decimal places.

$((4.1^2+2)\div((1.2-1)^3+20))\wedge 2 = \underline{0.8838}$

16. Let $h(x)=\sqrt{x}$, $g(x)=x-1$. Calculate $\left(\frac{h}{g}\right)(4) = \frac{h(4)}{g(4)} = \frac{\sqrt{4}}{4-1} = \underline{\frac{2}{3}}$

17. The points (5,-1) are and (2,3) are endpoints of the diameter of a circle. Find the center, the radius, and an equation of the circle.

$\text{dist}((5,-1),(2,3)) = \sqrt{(5-2)^2+(-1-3)^2} = \sqrt{9+16} = \sqrt{25} = 5$

Hence, radius $= 5/2 = 2.5$.

$\text{mid}((5,-1),(2,3)) = \left(\frac{5+2}{2}, \frac{-1+3}{2}\right) = \left(\frac{7}{2}, 1\right)$ ↙ Center

The equation:

$\underline{(x-\frac{7}{2})^2+(y-1)^2 = 2.5^2}$

18. Write in the form $a+bi$:

(a) $(2+i)(1-3i) = 2-6i+i+3 = \underline{5-5i}$

(b) $(1+i)-(1-2i) = 1+i-1+2i = \underline{0+3i}$