Practice Exam 2 - Solutions

1) $\sqrt{a^2+b^2}$ can't be simplified! Radical can't be distributed over addition. C

2) $\sqrt{\sqrt{\frac{x^7}{y^8}}} = \left(\left(\frac{x^7}{y^8}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}} = \left(\frac{x^7}{y^8}\right)^{\frac{1}{4}} = \frac{x^{\frac{7}{4}}}{y^{\frac{8}{4}}} = x^{\frac{7}{4}} \cdot y^{-2}$ A

3) $\sqrt{a^4b+a^4c} = \sqrt{a^4(b+c)} = \sqrt{a^4}\sqrt{b+c} = a^2\sqrt{b+c}$ B

4) $\frac{x^2}{\sqrt{x^2+4}} - \frac{\sqrt{x^2+4}\cdot\sqrt{x^2+4}}{\sqrt{x^2+4}} = \frac{x^2-(x^2+4)}{\sqrt{x^2+4}} = -\frac{4}{\sqrt{x^2+4}}$ A

5) $\sqrt{64c^8d^6} = 8c^4d^3$ B

6) $(a^9b^6)^{\frac{1}{3}} \cdot (ab^3)^{\frac{1}{2}} = a^{\frac{9}{3}} \cdot b^{\frac{6}{3}} \cdot a^{\frac{1}{2}} \cdot b^{\frac{3}{2}} = a^3 \cdot b^2 \cdot a^{\frac{1}{2}} \cdot b^{\frac{3}{2}} =$

$= a^{\frac{7}{2}} \cdot b^{\frac{7}{2}}$ B

7) (a) $\log_5\left(\frac{1}{25}\right) = -2$ as $5^{-2} = \frac{1}{25}$

(b) $\log_{10} .001 = -3$ as $10^{-3} = \frac{1}{1000}$

(c) $\ln e^{10} = 10$ as $e^{10} = e^{10}$

(d) $\log_{16}(4) = \frac{1}{2}$ as $16^{\frac{1}{2}} = 4$

(e) $\log_2(-8)$ is undefined as $2^x > 0$.

(f) $\log_a(a^x) = x$ as $a^x = a^x$.

8) (a) $2^{\log_2(8)} = 8$ from the def. of logarithm

(b) $e^{\ln x} = x$ " " "

(c) $3^{\log_3 7} = 7$ " " "

9) $f(x) = \dfrac{2x^2+1}{x^2-1}$

The numerator and the denominator are of the same degree. Hence,

$y = \frac{2}{1} = 2$ is a horizontal asymptote.

(Quotient of leading coefficients).

At $x = 1$ and $x = -1$ the denominator is 0 and the numerator is $\neq 0$.

Hence, $x = 1$ and $x = -1$ are vertical asymptotes.

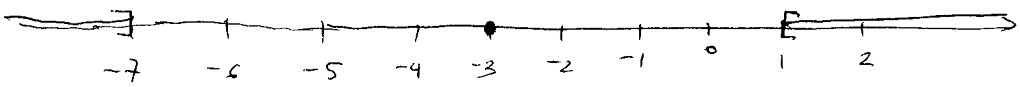
10) No. $f(x) = \dfrac{x^2-4}{x-2} = \dfrac{(x-2)(x+2)}{(x-2)} = x+2$. $f(x)$ coincides

with the linear function $y = x+2$ for all $x \neq 2$. It does not have

any asymptotes.

11) a) $|x+3| \geq 4$.

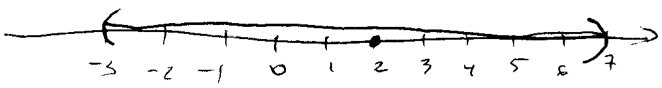
$|x+3|$ is the distance of x from -3. Hence, $|x+3| \geq 4$ if



$x \leq -7$ or $x \geq 1$

b) $|x-2| < 5$. The distance of x from 2 is less than 5

for $-3 < x < 7$



12) $x^2 + 4x + y^2 - 6y - 12 = x^2 + 4x + 4 - 4 + y^2 - 6y + 9 - 9 - 12 =$

$= (x+2)^2 + (y-3)^2 - 25$

Hence, the equation becomes

$$(x+2)^2 + (y-3)^2 = 25$$

Center: $(-2, 3)$ Radius: 5

13) $g(x) = x^2 - 7x + 12 = x^2 - 7x + \frac{49}{4} - \frac{49}{4} + 12 =$

$= \left(x - \frac{7}{2}\right)^2 - \frac{1}{4}$

Vertex: $\left(\frac{7}{2}, -\frac{1}{4}\right)$.

14) $x^2 + 10x + 23 = x^2 + 10x + 25 - 25 + 23 =$

$= (x + 5)^2 - 2$

The equation becomes

$(x+5)^2 = 2$

$x + 5 = \pm\sqrt{2}$ $\quad x_1 = -5 + \sqrt{2}, \quad x_2 = -5 - \sqrt{2}$

15) (a) $3x - 5 < 6 - 2x$

$5x < 11$

$x < \frac{11}{5}$

(b) $10x - 4 \leq 13 - 7x$

$17x \leq 17$

$x \leq 1$

16) $p(x) = x(x^2 - 3x + 2)$

Zeros of $x^2 - 3x + 2$ are 1 and 2. Hence, $p(x)$ factors as

$p(x) = x(x-1)(x-2)$.

17) No. Does not pass the horizontal line test.

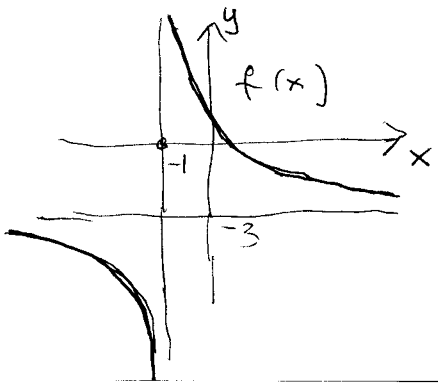
18) Yes. Does pass the horizontal line test.

19) According to our rules, we may take, for example,

$f(x) = \frac{2x+1}{x-3}$

20) $f(x) = \frac{1-3x}{x+1}$

Horizontal: $y = -3$ $\quad$ Vertical: $x = -1$



21) We graph the polynomial, use the trace and oblition on estimate:

$$x_1 = 1.8888 \quad , \quad x_2 = -0.3333 \quad , \quad x_3 = -1.5238$$

22)

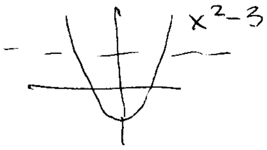
$$\begin{array}{r|l} & x^2 - 2x + 4 \\ \hline x+2 & x^3 + 0x^2 + 0\cdot x - 8 \\ & \underline{x^3 + 2x^2} \\ & \quad -2x^2 + 0\cdot x - 8 \\ & \quad \underline{-2x^2 - 4x} \\ & \qquad 4x - 8 \\ & \qquad \underline{4x + 8} \\ & \qquad\quad -16 \end{array}$$

$$\underline{P(x) = (x^2 - 2x + 4)(x+2) - 16}$$

23) (a) The function is linear, hence, one-to-one. The inverse exists.

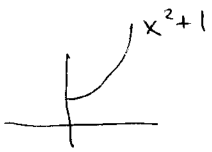
$y = 2x - 3 \qquad x = \frac{y+3}{2} = f^{-1}(y) \qquad \underline{f^{-1}(x) = \frac{x+3}{2}}$

(b)



Not one-to-one. No inverse.

(c) $f(x) = x^2 + 1 \quad , \quad x \geq 0$



one-to-one in this domain.

$y = x^2 + 1 \qquad , \qquad x = \sqrt{y-1} = f^{-1}(y)$

$$\underline{f^{-1}(x) = \sqrt{x-1}}$$

24) a) $f(x) = \frac{2}{x+3} \qquad y = \frac{2}{x+3} \quad , \quad yx + 3y = 2 \quad , \quad x = \frac{2-3y}{y} = f^{-1}(y)$

$$\underline{f^{-1}(x) = \frac{2-3x}{x}}$$

b) $f(x) = \frac{2+x}{x} \quad , \quad y = \frac{2+x}{x} \quad , \quad yx = 2 + x \quad , \quad x(y-1) = 2 ,$

$x = \frac{2}{y-1} = f^{-1}(y) \quad , \quad \underline{f^{-1}(x) = \frac{2}{x-1}}$

25) $f(g(x)) = f(\sqrt{x^2+1}) = 2\sqrt{x^2+1} - 3$

$g(f(x)) = g(2x-3) = \sqrt{(2x-3)^2+1}$

26) a) $h(x) = (x^3-2)^{10}$

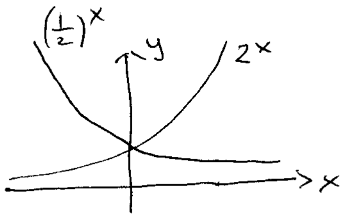
One way: $g(x) = x^3 - 2$, $f(x) = x^{10}$

Another: $g(x) = x^3$, $f(x) = (x-2)^{10}$

b) $h(x) = \dfrac{1}{\sqrt{x+1}}$

One way: $g(x) = \sqrt{x+1}$, $h(x) = \dfrac{1}{x}$

Another, $g(x) = x+1$, $h(x) = \dfrac{1}{\sqrt{x}}$

27) 

$(\frac{1}{2})^x = 2^{-x}$ so they are symmetric with respect to the y axis.

28) a) $\log_{10} 5 = 0.6989$ ("log" button)

b) $\ln(50) = 3.9120$ ("ln" button)

(c) $\log_2(7) = \dfrac{\ln 7}{\ln 2} = 2.8073$

(d) $\log_5(20) = \dfrac{\ln(20)}{\ln(5)} = 1.8613$