MTH 111 **EXAM III** **May 1, 2002**

Your name: **Your instructor:**

The exam consists of 15 problems. Problems 1-10 are worth 6 points each. Problems 11-15 are worth 8 points each. In all problems, you have to show your work and provide neat and clear explanations. Partial credit is possible.

1) Given that $\log_2(A) = 3$, $\log_2(B) = 4$ find the following logarithms:

(a) $\log_2(AB) = \log_2 A + \log_2 B = 3 + 4 = 7$

(b) $\log_2(\frac{A^2}{B}) = 2\log_2 A - \log_2 B = 2 \cdot 3 - 4 = 2$

(c) $\log_2(\sqrt{A}) = \frac{1}{2}\log_2 A = \frac{3}{2}$

2) Express as a single logarithm and simplify as far as possible

(a) $\frac{1}{2}\ln(z) + 5\ln(y) - 2\ln(y^3) = \ln\sqrt{z} + \ln y^5 - \ln y^6 = \ln\frac{\sqrt{z}\,y^5}{y^6} =$

$= \ln\frac{\sqrt{z}}{y}$

(b) $2\ln(\sqrt{x}) - \ln(x^2) = \ln x - \ln x^2 = \ln\frac{x}{x^2} = \ln\frac{1}{x}$

3) Express in terms of sums and differences of logarithms. Simplify as far as possible.

(a) $\ln(\frac{xy^3}{z^2+1}) = \ln(xy^3) - \ln(z^2+1) = \underline{\ln x + 3\ln y - \ln(z^2+1)}$

(b) $\log_a(\sqrt{xy}) = \frac{1}{2}\log_a(xy) = \underline{\frac{1}{2}\log_a x + \frac{1}{2}\log_a y}$

4) Simplify

(a) $\ln(e^{(x+1)}) = \underline{x+1}$

(b) $e^{(4\ln(x)-\ln(xy))} = e^{\ln x^4 - \ln(xy)} = e^{\ln\frac{x^4}{xy}} = \frac{x^4}{xy} = \underline{\frac{x^3}{y}}$

5) Solve for x.

(a) $\log_2(10+3x) = 3$

$$2^{\log_2(10+3x)} = 2^3$$

$$10 + 3x = 8$$

$$\underline{x = -\frac{2}{3}}$$

(b) $\log_2(x+1) + \log_2(x-1) = 3$

$$\log_2((x+1)(x-1)) = \log_2(x^2-1)$$

$$2^{\log_2(x^2-1)} = 2^3$$

$$x^2 - 1 = 8, \quad x^2 = 9, \quad x = \pm 3.$$

$x = -3$ is not a solution as $\log_2(-3-1)$ is undefined.

$\underline{x = 3}$ is the only solution to our equation.

(6) Solve for x. Give your solution to four decimal places.

$$3^x = 2^{(x-1)} \quad / \ln(\cdot)$$

$$\ln 3^x = \ln 2^{x-1}$$

$$x \ln 3 = (x-1)\ln 2, \quad x\ln 3 = x\ln 2 - \ln 2$$

$$x(\ln 3 - \ln 2) = -\ln 2, \quad x = \frac{-\ln 2}{\ln 3 - \ln 2} \cong \underline{-1.7095}$$

7) For each of the angles below give its exact radian measure in terms of π.

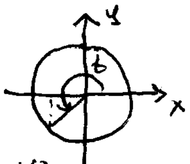
(a) $270° = 3 \cdot 90° = \frac{3\pi}{2}$

(b) $45° = \frac{\pi}{4}$

8) Suppose that the angle t lies in the III quadrant. Suppose that $\sin(t) = -0.4$. Find $\cos(t)$ and $\tan(t)$.

$\cos^2(t) = 1 - (-0.4)^2 = 0.84$. Hence, $\cos t = \sqrt{0.84}$ or $\cos t = -\sqrt{0.84}$

In the III quadrant $\cos(t)$ is negative, thus



$\cos(t) = -\sqrt{0.84} = -0.9165.$

$\tan(t) = \frac{\sin(t)}{\cos(t)} = \frac{-0.4}{-0.9165} = 0.4364$

$\cos(t) = x < 0$

9) Determine the amplitude and the period of the following functions

(a) $f(x) = -2\sin(\pi x)$

Ampl. $= |-2| = 2$

Period $= \frac{2\pi}{\pi} = 2$

(b) $f(x) = 3\cos(\frac{1}{2}x)$

Ampl $= 3$

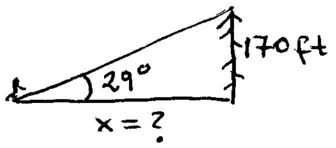
Period $= \frac{2\pi}{\frac{1}{2}} = 4\pi$

10) For each of the angles below give its exact measure in degrees

(a) $\frac{\pi}{3}$ $= 60°$ as $\pi = 180°$

(b) $\frac{\pi}{2}$ $= 90°$ as $\pi = 180°$

11) A forester measures that the elevation angle from the point he is standing to the top of a tree is 29°. The height of the tree is 170 ft. How far from the base of the tree is is the forester standing?



$\frac{170}{x} = \tan 29° = 0.5543$

$x = \frac{170}{0.5543} = \underline{306.69\text{ ft}}$

12) A radioactive element decays exponentially according to the formula

$$R(t) = 200e^{-kt} \quad ,$$

where the amount of the element, R, is measured in grams, the time t is measured in months. The initial amount of 200 grams at $t = 0$ decays to 170 grams after 3 months.

(a) Find the constant k.

(b) Find the half-life of the element.

(a) After 3 months, at $t = 3$ we have 170 g:

$200\,e^{-k\cdot 3} = 170$

Solving for k:

$e^{-3k} = \frac{170}{200}$ $\qquad -3k = \ln \frac{170}{200}$

$k = \frac{\ln\left(\frac{170}{200}\right)}{-3} \cong \underline{0.0542}$

(b) $t_h = ?$

$200\,e^{-0.0542\cdot t_h} = 100$

$e^{-0.0542\cdot t_h} = \frac{1}{2} \;/\ln(\cdot)$

$-0.0542\,t_h = \ln \frac{1}{2}$

$t_h = \frac{\ln \frac{1}{2}}{-0.0542} = \underline{12.78}$ months

13) The population of a town, $P(t)$, measured in thousands increases according to the formula

$$P(t) = 15(1.05)^t \quad ,$$

where t is the time in years after Jan.1, 1995.

(a) What will the population be on Jan.1, 2005?

(b) When will the population reach 40 thousand?

(a) Jan 1, 2005 corresponds to $t = 10$. Hence,

$P(10) = 15(1.05)^{10} = 24.433$ thousand people

(b) ? t $\quad 15(1.05)^t = 40 \quad , \quad (1.05)^t = \frac{40}{15} \;/ \ln(\cdot)$

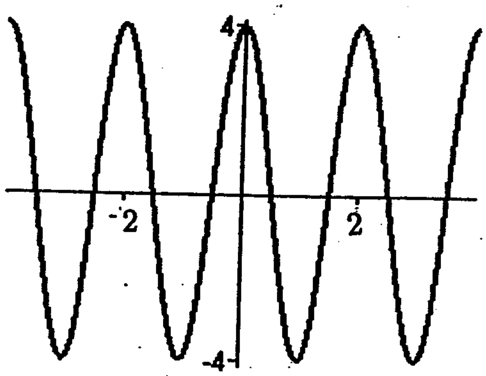
$\ln(1.05)^t = \ln\frac{40}{15} \quad , \quad t \ln 1.05 = \ln\frac{40}{15} \quad , \; t = \frac{\ln\frac{40}{15}}{\ln 1.05} \cong 20.1$ years.

The population will reach 40,000 during 2015.

14) Find a periodic function $f(x)$ whose amplitude is 2, period 4π and such that $f(0) = 0$.

$f(x) = 2\sin\left(\frac{1}{2}x\right)$

15) Find a possible formula for the graph below.



Amp $= 4$ $\quad$ Period $= \frac{2\pi}{|B|} = 2$

So $A = 4$, $B = \pi$

$f(x) = 4\cos(\pi x)$