MTH 111 **Exam 2** **April 1, 2002**

Your name:
Yours Instructor's Name:

Part I -- No calculators allowed.

Some of the problems below seem like multiple-choice problems, but indeed they are not. You have to explain your answers, no credit will be given without explanations. Partial credit is possible, similarly as on Exam I. Your explanations should consist of complete sentences and correct mathematical formulas.

1. Simplify

$$\sqrt{1+x^2}$$

(A) $1+x$ **(B)** $1+|x|$ **(C)** Can't be simplified **(D)** $1+\sqrt{x^2}$

The radical cannot be distributed over a sum. Hence, the expression can't be simplified.

2. Simplify.

$$\sqrt{\sqrt{\frac{x^5}{y^{16}}}}$$

(A) $x^{\left(\frac{5}{4}\right)} y^{(-2)}$ **(B)** $\frac{\sqrt{x^5}}{y^4}$ **(C)** $x^{\left(\frac{5}{4}\right)} y^{(-4)}$ **(D)** $x^{\left(\frac{5}{2}\right)} y^{(-4)}$

$$\sqrt{\sqrt{\frac{x^5}{y^{16}}}} = \left(\left(\frac{x^5}{y^{16}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}} = \left(\frac{x^5}{y^{16}}\right)^{\frac{1}{4}} = \frac{x^{\frac{5}{4}}}{y^{\frac{16}{4}}} = x^{\frac{5}{4}} \cdot y^{-4}$$

3. Simplify

$$\sqrt{8\,a^6\,b+8\,a^6\,c}$$

(A) $4\,a^4\,b+4\,a^4\,c$ **(B)** $4\,a^3\sqrt{b+c}$ **(C)** $2\,a^3\sqrt{2\,b}+3\,a^3\sqrt{2\,c}$ **(D)** $2\,a^3\sqrt{2\,b+2\,c}$

$\sqrt{8a^6b+8a^6c} = \sqrt{8a^6(b+c)} = \sqrt{8a^6}\cdot\sqrt{b+c} =$

$= 2\sqrt{2}\cdot\sqrt{a^6}\cdot\sqrt{b+c} = 2\sqrt{2}\,a^3\sqrt{b+c} =$

$= 2a^3\sqrt{2b+2c}$

4. Assume a>0, c>0. Simplify

$$(a^6\,c^2)^{\left(\frac{1}{3}\right)}\,(a\,c^2)^{\left(\frac{1}{2}\right)}$$

(A) $a^{\left(\frac{5}{2}\right)}\,c^{\left(\frac{5}{2}\right)}$ **(B)** $a^{\left(\frac{5}{2}\right)}\,c^{\left(\frac{5}{3}\right)}$ **(C)** $c^{\left(\frac{5}{3}\right)}\,a^3\sqrt{a}$ **(D)** None of the above

$(a^6c^2)^{\frac{1}{3}}\cdot(ac^2)^{\frac{1}{2}} = a^{\frac{6}{3}}c^{\frac{2}{3}}\cdot a^{\frac{1}{2}}\cdot c^{\frac{2}{2}} =$

$= a^2c^{\frac{2}{3}}\cdot a^{\frac{1}{2}}\cdot c = \underline{a^{\frac{5}{2}}\cdot c^{\frac{5}{3}}}$

5. Find the following logarithms, if defined. If not, write "undefined".

(a) $\log_3\left(\frac{1}{9}\right) = -2$ as $3^{-2} = \frac{1}{9}$

(b) $\log_{10}(1000) =$ 3 as $10^3 = 1000$

(c) $\ln(e^{10}) =$ 10 as $\ln e^x = x$

(d) $\log_9(3) =$ $\frac{1}{2}$ as $9^{\frac{1}{2}} = 3$

(e) $\log_5(-25) =$ undefined as $5^x > 0$ always.

6. Simplify

(a) $3^{\log_3(9)} =$ 9 as $a^{\log_a x} = x$

(b) $e^{\ln(x)} =$ x by the very formula

(c) $\log_a(a^x) =$ x as $a^x = a^x$

7. Without using your grapher, find all vertical and horizontal asymptotes of $f(x) = \dfrac{3x+1}{x+2}$.

Using rules of finding asymptotes, the asymptotes are

$y = 3$ (polynomial of the same degree, quotient of leading coeff. $= \frac{3}{1} = 3$

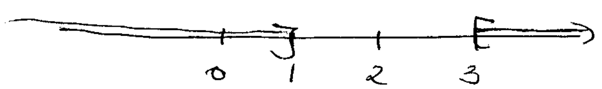
$x = -2$ (the denominator is 0 at $x=-2$, the numerator is $\neq$ from 0 at $x = -2$.)

8. Solve the following inequalities. Graph your solutions on the number line.

(a) $1 \le |x-2|$

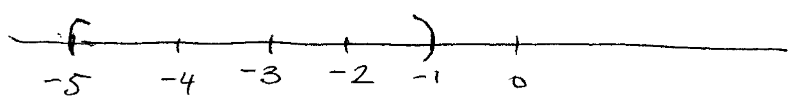
(b) $|x+3| < 2$

a)



$|x-2|$ is the distance of x from 2. The distance is greater or equal 1 if $x \geq 3$ or $x \leq 1$.

b)



$-5 < x < -1$

11. Solve the following inequalities:

(a) $4x - \frac{1}{2} < 6 - \frac{3x}{2}$ (b) $3x + 4 \le 5 + 7x$

(a) $8x - 1 < 12 - 3x$

$11x < 13$

$x < \frac{13}{11}$

(b) $-1 \le 4x$

$x \geq -\frac{1}{4}$

12. Factor completely into polynomials of degree one

$$p(x) = x^3 + x^2 - 6x.$$

$p(x) = x(x^2 + x - 6) = x(x-2)(x+3)$

$x^2 + x - 6 = 0$

$x_1 = 2, \quad x_2 = -3$

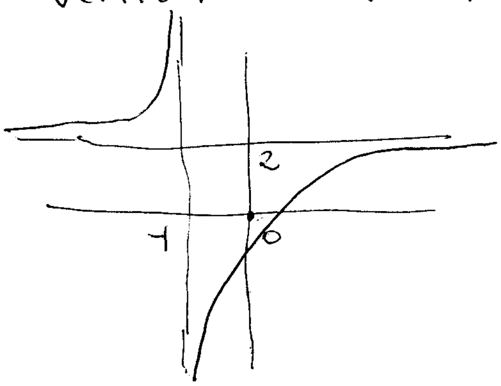
13. For the function

$$f(x) = \frac{2x - 1}{x + 1}$$

find all horizontal and vertical asymptotes. Graph the function.

Horizontal: $y = 2$

Vertical: $x = -1$ (From rules for finding asymptotes)



MTH 111 **Exam 2** **April 3, 2002**

Your name:
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Part II -- **You can use your calculators if you wish. In all problems, your explanations should consist of complete sentences and correct mathematical formulas. Each problem in this part is worth 6 points.**

9. Find the center, the radius, and the equation in the standard form for the circle

$$x^2 - 8x + y^2 - 3y + 12 = 0 .$$

HINT: Complete the square in x terms and in y terms.

$$x^2 - 8x + 16 - 16 + y^2 - 3y + \frac{9}{4} - \frac{9}{4} + 12 = 0$$

$$(x-4)^2 + \left(y - \frac{3}{2}\right)^2 = 4 + \frac{9}{4}$$

$$(x-4)^2 + \left(y - \frac{3}{2}\right)^2 = \frac{25}{4}$$

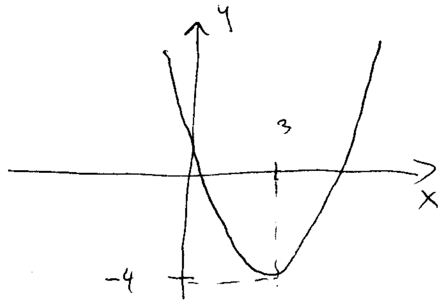
Center: $\left(4, \frac{3}{2}\right)$

Radius: $\frac{5}{2}$

10. By completing the square, find the vertex of the parabola $g(x) = x^2 - 6x + 5$. Sketch the parabola.

$$g(x) = x^2 - 6x + 9 - 9 + 5 = (x-3)^2 - 4$$

vertex: $(3, -4)$



14. Let $P(x) = x^4 + 9x^2 + 20$, $d(x) = x^2 + 4$. Divide P(x) by d(x) using **long division**. Write P(x) in terms of the divisor, the quotient, and the remainder.

$$
\begin{array}{r|l}
 & x^2 \quad\quad\quad + 5 \\
\hline
x^2+4 & x^4 + 0\cdot x^3 + 9x^2 + 0x + 20 \\
 & x^4 \quad\quad\quad + 4x^2 \\
\hline
 & \quad\quad\quad 5x^2 + 0x + 20 \\
 & \quad\quad\quad 5x^2 \quad\quad + 20 \\
\hline
 & \quad\quad\quad = \quad = \quad =
\end{array}
$$

$Q(x) = (x^2+5)$, $R(x) = 0$

$P(x) = (x^2+4)(x^2+5) + 0$

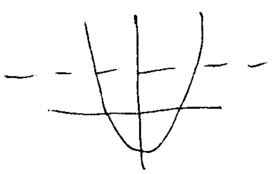
15. For the following functions *f(x)*, find the inverse $f^{(-1)}(x)$ if the inverse exists. If the inverse does not exist, explain why.

(a) $f(x) = 3x - 2$, domain: all x. $f(x)$ is linear, thus, one-to-one. $f^{-1}(x)$ exists.

$y = 3x - 2$ $\quad x = \frac{y+2}{3} = f^{-1}(y)$

$f^{-1}(x) = \frac{x+2}{3}$

(b) $f(x) = 2x^2 - 1$, domain: all x



Not one-to-one. No inverse.

(c) $f(x) = \frac{x+4}{x-3}$, domain: all x different from 3. One-to-one (Graph on your grapher)

$y = \frac{x+4}{x-3}$

$yx - 3y = x + 4$

$x(y-1) = 4 + 3y$

$x = \frac{4+3y}{y-1} = f^{-1}(y)$

$f^{-1}(x) = \frac{3x+4}{x-1}$

16. Let $f(x) = 4 - 3x$, $g(x) = \sqrt{x^3 - 2}$. Find the composite functions f(g(x)) and g(f(x)). You don't have to simplify anything.

$f(g(x)) = f(\sqrt{x^3-2}) = 4 - 3\sqrt{x^3-2}$

$g(f(x)) = g(4-3x) = \sqrt{(4-3x)^3-2}$

17. For each function h(x) below, find at least two ways of representing h(x) as a composition h(x)=f(g(x)):

(a) $h(x) = (x^2 + 5)^{20}$

One way: $g(x) = x^2+5$, $f(x) = x^{20}$

Another: $g(x) = x^2$, $f(x) = (x+5)^{20}$

(b) $h(x) = \dfrac{1}{(x+1)^{10}}$

One way: $g(x) = x+1$, $f(x) = \dfrac{1}{x^{10}}$

Another: $g(x) = (x+1)^{10}$, $f(x) = \dfrac{1}{x}$

18. Using your calculator, find

(a) $\log_{10}(3) =$ 0.4771 ("log" button)

(b) $\ln(20) =$ 2.9957 ("ln" button)

(c) $\log_3(8) = \dfrac{\ln 8}{\ln 3} = 1.8927$

(d) $\log_5(15) = \dfrac{\ln 15}{\ln 5} = 1.6826$