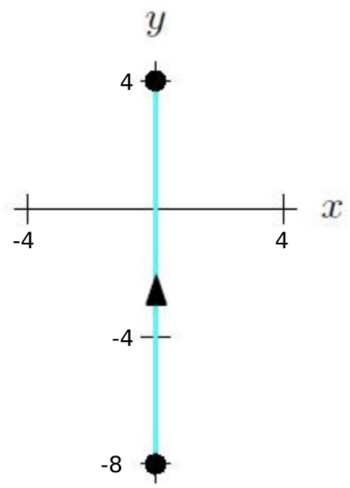
Class Worksheet 4/7/22 - Solutions

Example 1:

Find the parameterization for the curve shown, where the orientation shown corresponds to increasing values of t.



- ○ $x = 0, y = -t, -8 \leq t \leq 4$
- ○ $x = 0, y = t, -8 \leq t \leq 4$
- ○ $x = t, y = 0, -8 \leq t \leq 4$
- ○ $x = 1, y = t, -8 \leq t \leq 4$
- ○ $x = t, y = t, -8 \leq t \leq 4$

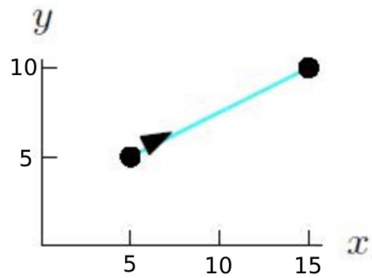
Solution

Since we are moving on the y-axis, $x = 0$ and y goes from -8 to 4. Thus, a parameterization is

$$x = 0, \quad y = t, \quad -8 \leq t \leq 4.$$

Example 2:

Find the parameterization for the curve shown, where the orientation shown corresponds to increasing values of t.



- ○ $x = 3t, y = 2t, 0 \leq t \leq 5$
- ○ $x = 5 + 3t, y = 5 + 2t, 0 \leq t \leq 5$
- ○ $x = 10 + t, y = 5 + t, 0 \leq t \leq 5$
- ○ $x = 5 + t, y = 5 + t, 0 \leq t \leq 10$
- ○ $x = 5 + 10t, y = 5 + 5t, 0 \leq t \leq 1$

Solution

We want the straight line segment from $(5, 5)$ to $(15, 10)$. The position vector of $(5, 5)$ is $5\vec{i} + 5\vec{j}$ and the displacement vector from $(5, 5)$ to $(15, 10)$ is $10\vec{i} + 5\vec{j}$, so the line has equation

$$\vec{r} = 5\vec{i} + 5\vec{j} + t\left(10\vec{i} + 5\vec{j}\right),$$

or

$$x = 5 + 10t, \quad y = 5 + 5t.$$

This passes $(5, 5)$ when $t = 0$ and $(15, 10)$ when $t = 1$, so a parameterization of the line is

$$x = 5 + 10t, \quad y = 5 + 5t, \quad 0 \leq t \leq 1.$$

Example 3:

Find parametric equations in terms of t for the line in the direction of the vector $5\vec{j} + 2\vec{k}$ and through the point $(5, -1, 1)$.

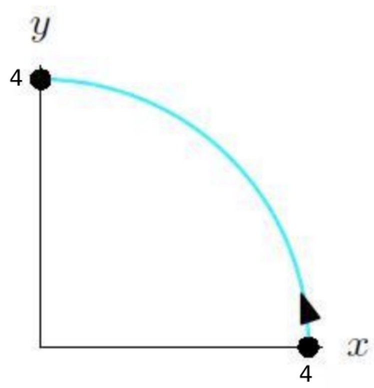
Solution:
There are infinite number of possible paramaterizations for this line. One possible parameterization is

$$x = 5, y = -1 + 5t, z = 1 + 2t,$$

coming from the vector equation $\vec{r}(t) = 5\vec{i} - \vec{j} + \vec{k} + t(5\vec{j} + 2\vec{k})$.

Example 4:

Find the parameterization for the curve shown, where the orientation shown corresponds to increasing values of t.



- $x = 4\cos t, y = 4\sin t, 0 \le t \le \frac{\pi}{2}$
- $x = 16\cos t, y = 16\sin t, 0 \le t \le \frac{\pi}{2}$
- $x = 4\cos t, y = 4\sin t, 0 \le t \le \pi$
- $x = 16\sin t, y = 16\cos t, 0 \le t \le \pi$
- $x = 4\sin t, y = 4\cos t, 0 \le t \le \frac{\pi}{2}$

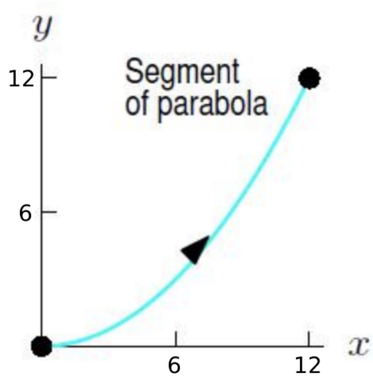
Solution

We want a quarter-circle of radius 4 starting at $(4, 0)$ and ending at $(0, 4)$. The equations $x = 4\cos t$, $y = 4\sin t$ describe counterclockwise motion in a circle of radius 4 centered at the origin, passing $(4, 0)$ when $t = 0$ and $(0, 4)$ when $t = \frac{\pi}{2}$. So, a parameterization of the curve is

$$x = 4\cos t, \quad y = 4\sin t, \quad 0 \le t \le \frac{\pi}{2}.$$

Example 5:

Find the parameterization for the curve shown, where the orientation shown corresponds to increasing values of t.



- $x = 6t, y = \frac{1}{12}t^2, 0 \le t \le 6$
- $x = 2t, y = t^2, 0 \le t \le 6$
- $x = t, y = t^2, 0 \le t \le 12$
- $x = t, y = \frac{1}{12}t^2, 0 \le t \le 12$
- $x = 2t, y = \frac{1}{12}t^2, 0 \le t \le 12$

Solution

The curve is a segment of a parabola $y = ax^2$ starting at $(0, 0)$ and ending up at $(12, 12)$. Thus the parabola has equation $y = \frac{1}{12}x^2$. Since the x goes from 0 to 12, a parameterization of the curve is

$$x = t, \quad y = \frac{1}{12}t^2, \quad 0 \le t \le 12.$$