Class Worksheet 4/5/22 - Solutions

Example 1:

Find an equation for the top half of the sphere $x^2 + y^2 + z^2 = 25$ in cylindrical coordinates.
NOTE: Enter the exact answer.

$z =$ ______

Solution:

The top half of the sphere has equation $z = \sqrt{25 - x^2 - y^2} = \sqrt{25 - r^2}$.

Example 2:

Evaluate the triple integral of $f(x, y, z) = \sin(x^2 + y^2)$ in cylindrical coordinates over the region W given by the solid cylinder with height 8 and with base of radius 5 centered on the z-axis at $z = -1$.
NOTE: Enter exact answers.
Enter the integral in the order $dr\, d\theta\, dz$.

Solution

Solution:

$$\begin{aligned}
\int_W f\, dV &= \int_{-1}^{7} \int_{0}^{2\pi} \int_{0}^{5} \left(\sin\left(r^2\right)\right) r\, dr\, d\theta\, dz \\
&= \int_{-1}^{7} \int_{0}^{2\pi} \left(-\frac{1}{2}\cos r^2\right)\Big|_0^5 \, d\theta\, dz \\
&= -\frac{1}{2} \int_{-1}^{7} \int_{0}^{2\pi} (\cos 25 - \cos 0)\, d\theta\, dz \\
&= -\pi \int_{-1}^{7} (\cos 25 - 1)\, dz \\
&= -8\pi(\cos 25 - 1) \\
&= 8\pi(1 - \cos 25)
\end{aligned}$$

Example 3:

Evaluate the triple integral of $f(x, y, z) = \dfrac{1}{\sqrt{x^2 + y^2 + z^2}}$ in spherical coordinates over the bottom half of the sphere of radius 10 centered at the origin.

NOTE: Enter exact answers.
Enter the integral in the order $d\phi\, d\theta\, d\rho$.

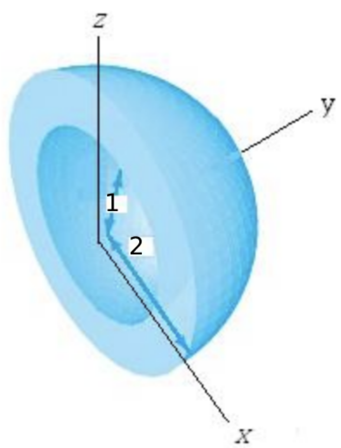
Solution:

We have:

$$\begin{aligned}
\int_W f\, dV &= \int_0^{10} \int_0^{2\pi} \int_{\pi/2}^{\pi} \frac{1}{\rho} \cdot \rho^2 \sin(\phi)\, d\phi\, d\theta\, d\rho \\
&= \int_0^{10} \int_0^{2\pi} \int_{\pi/2}^{\pi} \rho \sin(\phi)\, d\phi\, d\theta\, d\rho \\
&= \int_0^{10} \int_0^{2\pi} \rho\, d\theta\, d\rho \\
&= 2\pi \int_0^{10} \rho\, d\rho \\
&= 100\pi
\end{aligned}$$

Example 4:

Which of the following represents the integral of the function f over the given region?



- $\int_0^{\pi}\int_0^{\pi}\int_1^{2} f\cdot r^2\sin\theta\,dr\,dz\,d\theta$
- $\int_0^{2\pi}\int_0^{2\pi}\int_1^{2} f\cdot \rho^2\sin\phi\,d\rho\;d\phi\;d\theta$
- $\int_0^{2\pi}\int_0^{2\pi}\int_1^{2} f\cdot r\,dr\,dz\,d\theta$
- $\int_1^{2}\int_1^{2}\int_1^{2} f\,dx\,dy\,dz$
- $\int_0^{\pi}\int_0^{\pi}\int_1^{2} f\cdot \rho^2\sin\phi\,d\rho\,d\phi\,d\theta$

Solution

Using spherical coordinates, we get:

$$\int_0^{\pi}\int_0^{\pi}\int_1^{2} f\cdot \rho^2\sin\phi\,d\rho\,d\phi\,d\theta.$$