Class Worksheet 4/14/22 Solutions

Example 1:

Find the velocity and acceleration vectors for the parametric equations $x = 2t,\ y = 5t^2,\ z = 5t^3$.

NOTE: Enter an answer in each field.

$\vec{v} =$ [ ] $\vec{i} +$ [ ] $\vec{j} +$ [ ] $\vec{k}$

$\vec{a} =$ [ ] $\vec{i} +$ [ ] $\vec{j} +$ [ ] $\vec{k}$

Solution:

The velocity vector $\vec{v}$ is given by:

$$\begin{aligned} \vec{v} &= \frac{d(2t)}{dt}\vec{i} + \frac{d(5t^2)}{dt}\vec{j} + \frac{d(5t^3)}{dt}\vec{k} \\ &= 2\vec{i} + 10t\vec{j} + 15t^2\vec{k} \end{aligned}$$

The acceleration vector $\vec{a}$ is given by:

$$\begin{aligned} \vec{a} &= \frac{d(2)}{dt}\vec{i} + \frac{d(10t)}{dt}\vec{j} + \frac{d(15t^2)}{dt}\vec{k} \\ &= 0\vec{i} + 10\vec{j} + 30t\vec{k} \\ &= 10\vec{j} + 30t\vec{k} \end{aligned}$$

Example 2:

Find the velocity $\vec{v}$ and speed $\|\vec{v}\|$. Find any times at which the particle stops.

$$x = 2t,\ y = 2t^2,\ z = 3t^3$$

$\vec{v} =$ [ ] $\vec{i} +$ [ ] $\vec{j} +$ [ ] $\vec{k}$

$\|\vec{v}\| =$ [ ]

Solution:
The velocity vector $\vec{v}$ is given by:

$$\begin{aligned} \vec{v} &= \frac{d}{dt}(2t)\,\vec{i} + \frac{d}{dt}(2t^2)\,\vec{j} + \frac{d}{dt}(3t^3)\,\vec{k} \\ &= 2\,\vec{i} + 4t\,\vec{j} + 9t^2\,\vec{k} \end{aligned}$$

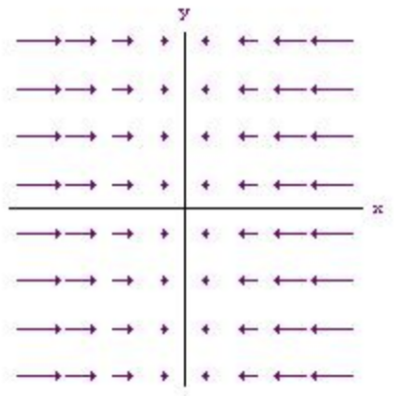
The speed is given by:

$$\|\vec{v}\| = \sqrt{4 + 16t^2 + 81t^4}$$

$\|\vec{v}\|$ is never zero since $4 + 16t^2 + 81t^4 \geq 4$ for all t. Thus, the particle never stops.

Example 3:

Choose the correct formula for the vector field.



- $\vec{F}\ (x, y) = y\ \vec{i}$
- $\vec{F}\ (x, y) = -x\ \vec{j}$
- $\vec{F}\ (x, y) = -x\ \vec{i}$
- $\vec{F}\ (x, y) = x\ \vec{i}$
- $\vec{F}\ (x, y) = x\ \vec{j}$

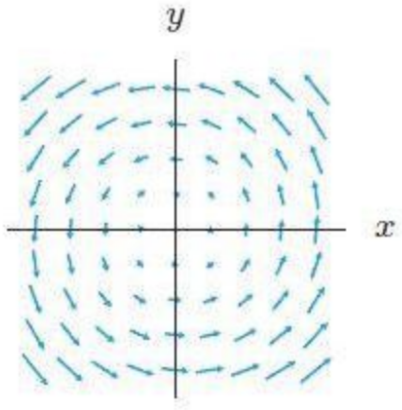
Solution

We see that the vectors in the field are constant along vertical lines, so the vector field does not depend on y. Also we can note that the vectors are parallel to the x-direction, pointing to the left when x is positive and to the right when x is negative. The larger $|x|$ is, the longer the vector is.

Thus a formula is $\vec{F}\ (x, y) = -x\ \vec{\mathrm{i}}$.

Example 4:

Find a formula for the given vector field.



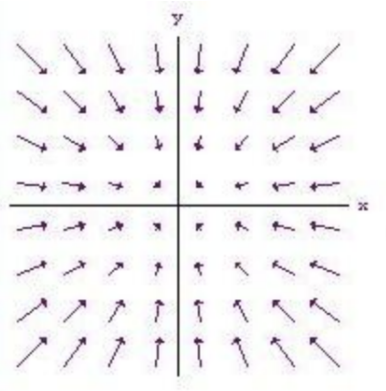
- $\vec{V} = x\,\vec{i}$
- $\vec{V} = -y\,\vec{i} + x\,\vec{j}$
- $\vec{V} = -y\,\vec{i}$
- $\vec{V} = x\,\vec{i} + y\,\vec{j} = \vec{r}$
- $\vec{V} = -x\,\vec{i} - y\,\vec{j} = -\vec{r}$

Solution

$\vec{V} = -y\,\vec{i} + x\,\vec{j}$

Example 5:

Choose the correct formula for the vector field.



- $\vec{F}\ (x, y) = x\ \vec{i}\ + y\ \vec{j}$
- $\vec{F}\ (x, y) = x\ \vec{i}\ - y\ \vec{j}$
- $\vec{F}\ (x, y) = -x\ \vec{i}\ + y\ \vec{j}$
- $\vec{F}\ (x, y) = x\ \vec{i}$
- $\vec{F}\ (x, y) = -x\ \vec{i}\ - y\ \vec{j}$

Solution

We see that the vectors of the given field are parallel to the direction of the negative position vector $-\vec{r} = -x\vec{i} - y\vec{j}$, and the larger $|r|$ is, the longer the vector of the field is.

Thus a formula is $\vec{F}\ (x, y) = -x\ \vec{i}\ - y\ \vec{j}$.

Example 6:

Consider the vector field $\vec{F} = \text{grad}\left(x^4 + e^{8y}\right)$.

Assume $x, y > 0$ and decide if

(a) The vector field is parallel to the x-axis, parallel to the y-axis, or neither.

The vector field is parallel to _____

(b) As x increases, the length increases, decreases, or neither.

As x increases, the length _____ .

(c) As y increases, the length increases, decreases, or neither.

As y increases, the length _____ .

Solution

(a) Since $\text{grad}\left(x^4 + e^{8y}\right) = 4x^3\ \vec{i}\ + 8e^{8y}\ \vec{j}$, the vector field is parallel to neither axis.

(b) The length increases as x increases.

(c) The length increases as y increases.