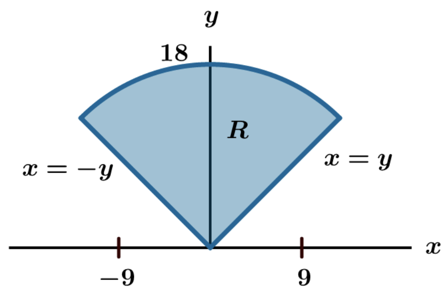
Class Worksheet 3/31/22 - Solutions

Example 1:

Write $\int_R f\, dA$ as an iterated integral in polar coordinates.
NOTE: Enter exact answers.



$$\int_R f\, dA = \int_{\square}^{\square} \int_{\square}^{\square} f(r,\theta)\ r\, dr\, d\theta$$

Solution:

$$\boxed{\int_{\pi/4}^{3\pi/4} \int_{0}^{18} f(r,\theta)\ r\, dr\, d\theta}$$

Example 2:

Convert the integral $\int_{-5}^{0}\int_{-\sqrt{25-x^2}}^{\sqrt{25-x^2}} x\,dy\,dx$ to polar coordinates and evaluate.

NOTE: Enter exact answers.

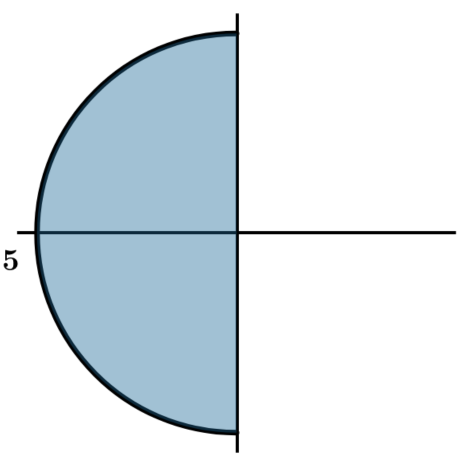
The integral in polar coordinates is

$$\int_{\square}^{\square}\int_{\square}^{\square} \square \; dr\,d\theta$$

NOTE: Enter the exact answer, or round to three decimal places.

Solution:

By the given limits $-5 \le x \le 0$, and $\sqrt{-25 - x^2} \le y \le \sqrt{25 - x^2}$, the region of integration is shown in the figure.



In polar coordinates, we have:

$$\int_{\pi/2}^{3\pi/2}\int_{0}^{5} r(\cos\theta)\,r\,dr\,d\theta = \int_{\pi/2}^{3\pi/2} \cos\theta \cdot \left(\frac{r^3}{3}\right)\Bigg|_0^5 d\theta$$

$$= \frac{125}{3}\int_{\pi/2}^{3\pi/2} \cos\theta\,d\theta$$

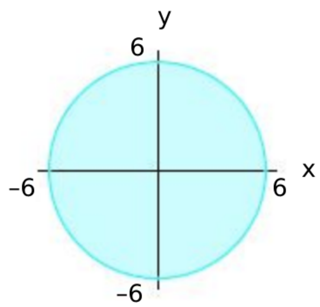
$$= \frac{125}{3}\sin\theta\Bigg|_{\pi/2}^{3\pi/2}$$

$$= \frac{125}{3}(-1-1)$$

$$= -\frac{250}{3}$$

Example 3:

Which of the following represents the integral of the function $f(x, y)$ over the given region?



- $\int_0^{2\pi}\int_0^6 f(r\cos\theta, r\sin\theta)\, r\, dr\, d\theta$
- $\int_{-6}^{6}\int_{-6}^{6} f(x, y)\, dx\, dy$
- $\int_0^{2\pi}\int_{-6}^{6} f(r\cos\theta, r\sin\theta)\, dr\, d\theta$
- $\int_0^{2\pi}\int_0^6 f(r\cos\theta, r\sin\theta)\, dr\, d\theta$
- $\int_0^{\pi}\int_{-6}^{6} f(x, y)\, dy\, dx$

<u>Solution</u>

A circle is best described in polar coordinates. The radius is 6, so r goes from 0 to 6. To include the entire circle, we need θ to go from 0 to 2π. The integral is

$$\int_0^{2\pi}\int_0^6 f(r\cos\theta,\ r\sin\theta)\, r\, dr\, d\theta.$$