Class Worksheet 3/29/22 - Solutions

Example 1:

Find the triple integral of the function

$f(x, y, z) = e^{-x-y-z}$

over the region W given by the rectangular box with corners at

$(0, 0, 0), (5, 0, 0), (0, 6, 0)$, and $(0, 0, 7)$.

Round your answer to two decimal places.

$\int_W f\, dV =$

Solution

$$\begin{aligned}
\int_W f\, dV &= \int_0^5 \int_0^6 \int_0^7 e^{-x-y-z}\, dz\, dy\, dx \\
&= \int_0^5 \int_0^6 \int_0^7 e^{-x} e^{-y} e^{-z}\, dz\, dy\, dx \\
&= \int_0^5 \int_0^6 e^{-x} e^{-y} \left(-e^{-z} \right) \Big|_0^7 \, dy\, dx \\
&= \int_0^5 \int_0^6 e^{-x} e^{-y} \left(-e^{-7} + 1 \right) dy\, dx \\
&= \left(-e^{-7} + 1 \right) \int_0^5 e^{-x} \left(-e^{-y} \right) \Big|_0^6 \, dx \\
&= \left(-e^{-6} + 1 \right) \left(-e^{-7} + 1 \right) \int_0^5 e^{-x}\, dx \\
&= \left(1 - e^{-5} \right) \left(1 - e^{-6} \right) \left(1 - e^{-7} \right) \\
&= 0.99
\end{aligned}$$

Example 2:

Find the volume of the region in the first octant bounded by the coordinate planes and the surface $x + y + z = 2$.

Enter the exact answer.

$V =$

Solution

We have $z = 2 - x - y$, and the region under the surface in the xy-plane is bounded by the axes and the line $x + y = 2$ or $y = 2 - x$. Thus the volume is given by

$$
\begin{aligned}
V &= \int_0^2 \int_0^{2-x} \int_0^{2-x-y} dz\,dy\,dx \\
&= \int_0^2 \int_0^{2-x} (2 - x - y)\,dy\,dx \\
&= \int_0^2 \left(2y - xy - \frac{y^2}{2} \Big|_0^{2-x} \right) dx \\
&= \int_0^2 \left(2(2-x) - x(2-x) - \frac{(2-x)^2}{2} \right) dx \\
&= \int_0^2 \left(\frac{x^2}{2} - 2x + 2 \right) dx \\
&= \left(\frac{x^3}{6} - \frac{2}{2}x^2 + 2x \right) \Big|_0^2 \\
&= \frac{4}{3}
\end{aligned}
$$