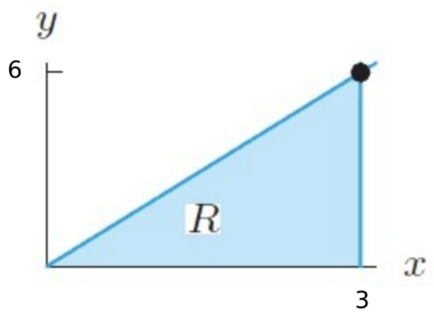
Class Worksheet 3/24/22

Example 1:

Integrate $f(x, y) = xy$ over the region R.



Enter the exact answer.

$$\int_R f(x, y)dA =$$

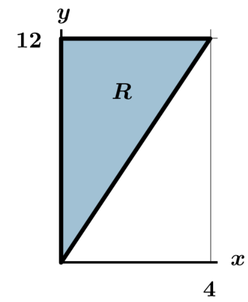
Solution

The diagonal line has equation $y = 2x$. Integrating with respect to y first gives

$$\begin{aligned}
\int_0^3 \int_0^{2x} xy\,dy\,dx &= \int_0^3 x\frac{y^2}{2}\bigg|_0^{2x} dx \\
&= \int_0^3 2x^3\,dx \\
&= \frac{x^4}{2}\bigg|_0^3 \\
&= \frac{81}{2}.
\end{aligned}$$

Example 2:

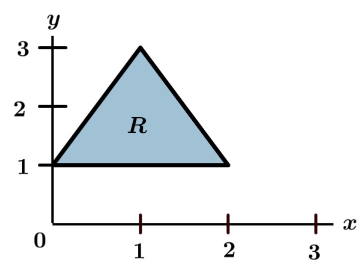
Write $\int_R f\,dA$ as an iterated integral for the shaded region R.



$$\int_0^4 \int_{3x}^{12} f(x,y)\,dy\,dx \qquad \text{or} \qquad \int_0^{12} \int_0^{y/3} f(x,y)\,dx\,dy$$

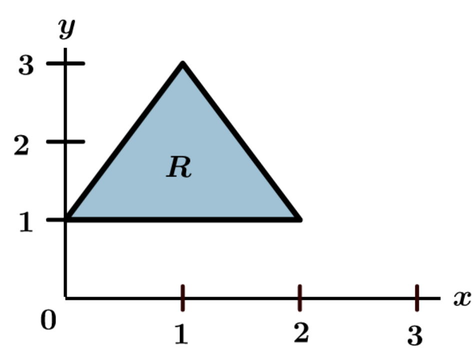
Example 3:

Write $\int_R f\,dA$ as an iterated integral for the shaded region R.



Solution:

The line on the left, through points $(0,1)$ and $(1,3)$, is the line $y = 2x+1$ or $x = \dfrac{y-1}{2}$. The line on the right, through points $(1,3)$ and $(2,1)$ is the line $y = -2x+5$ or $x = -\dfrac{y-5}{2}$. One way to set up this iterated integral is:



$$\int_1^3 \int_{(y-1)/2}^{-(y-5)/2} f(x,y)\,dx\,dy$$

The other option for setting up this integral requires two separate integrals, as follows:

$$\int_0^1 \int_1^{2x+1} f(x,y)\,dy\,dx + \int_1^2 \int_1^{-2x+5} f(x,y)\,dy\,dx$$