Class Worksheet 3/22/22

Example 1:

Evaluate the integral $\int_0^1 \int_0^4 (x^4 y)\,dy\,dx.$

Enter the exact answer.

$$\int_0^1 \int_0^4 (x^4 y)\,dy\,dx =$$

Solution

We evaluate the inside integral first:

$$\int_0^4 (x^4 y)\,dy = \left(\frac{x^4 y^2}{2}\right)\Big|_{y=0}^{y=4} = 8x^4.$$

Therefore, we have

$$\int_0^1 \int_0^4 (x^4 y)\,dy\,dx = \int_0^1 (8x^4)\,dx = \left(\frac{8x^5}{5}\right)\Big|_0^1 = \frac{8}{5}.$$

Example 2:

Evaluate the integral $\int_0^1 \int_0^1 ye^{xy}\,dx\,dy$.

NOTE: Enter the exact answer, or round to three decimal places..

$$\int_0^1 \int_0^1 ye^{xy}\,dx\,dy = \boxed{}$$

Solution:

Calculating the inner integral first, we have:

$$\begin{aligned}\int_0^1 \int_0^1 ye^{xy}\,dx\,dy &= \int_0^1 e^{xy}\Big|_0^1\,dy \\ &= \int_0^1 \left(e^y - e^0\right)dy \\ &= \int_0^1 (e^y - 1)\,dy = (e^y - y)\Big|_0^1 = e^1 - 1 - \left(e^0 - 0\right) = e - 2\end{aligned}$$

Example 3:

Let D be the region inside the unit circle centered at the origin, and let B be the bottom half of D. Decide (without calculation) whether the integral $\int_B 9x\,dA$ is positive, negative, or zero.

Solution

The function being integrated is $f(x, y) = 9x$, which is an odd function in x. Since B is symmetric with respect to x, the contributions to the integral cancel out, as $f(x, y) = -f(-x, y)$. Thus, the integral is zero.