Class Worksheet 3/10/22

Example 1:

Find the critical point(s) and classify them as local maxima, local minima, or saddle points.

$$f(x, y) = 200 - 3x^2 - 4x + 2xy - 5y^2 + 48y$$

Solution:
Setting $f_x = 0$ and $f_y = 0$ to find the critical point, we have

$$2y - 6x = 4 \qquad \text{and} \qquad 10y - 2x = 48$$

Solving these equations simultaneously gives $x = 1$ and $y = 5$.

Since $f_{xx} = -6, f_{yy} = -10$ and $f_{xy} = 2$ for all (x, y), at $(1, 5)$ the discriminant

$$D = (f_{xx})(f_{yy}) - (f_{xy})^2 = (-6)(-10) - (2)^2 = 56 > 0,$$

and $f_{xx} < 0$.

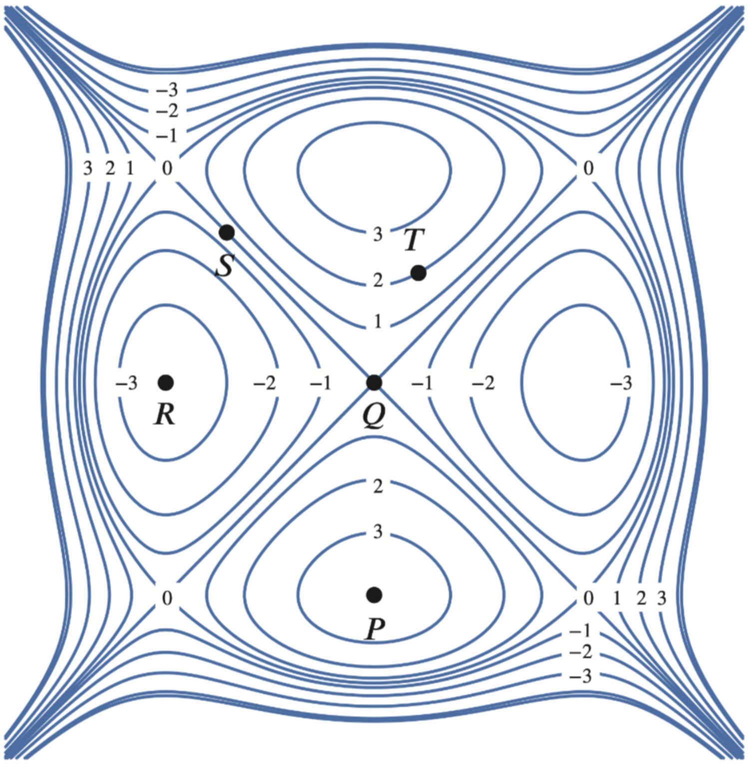
Thus $f(x, y)$ has a local maximum value at $(1, 5)$.

Example 2:

The function $g(x, y) = x^2 + y^3$ has a critical point at $(0, 0)$.
What sort of critical point is it?

Solution:
At the origin $g(0, 0) = 0$. Since $y^3 \geq 0$ for $y > 0$ and $y^3 < 0$ for $y < 0$, the function g takes on both positive and negative values near the origin, which must therefore be a saddle point. The second derivative test does not tell you anything since $D = 0$.

Example 3: Which of the points P, Q, R, S appear to be critical points? Classify those which are.



P is a local maximum;

Q is a saddle point;

R is a local minimum;

S is not a critical point as the graph near S seems to be a non-horizontal plane.