Class Worksheet 2/8/2022 - Solutions

Example 1: Let $f(x, y, z) = x^2 + 2x + y^2 + z^2$. Describe level surfaces of the function corresponding to function values $c = 3, 8, 15$.

In the formula for $f(x,y,z)$, complete the square in x terms:

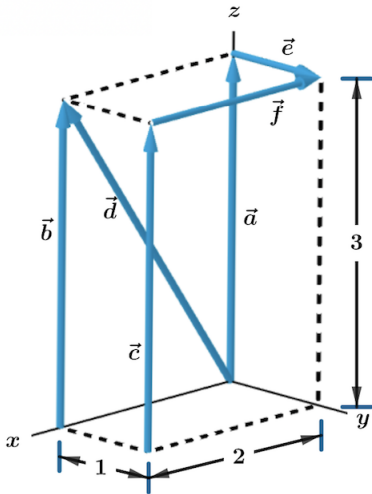$f(x,y,z) = (x^2 + 2x + 1) - 1 + y^2 + z^2 = (x+1)^2 + y^2 + z^2 - 1$

For any function value c, the level surface $f(x,y,z) = c$ is:

$(x+1)^2 + y^2 + z^2 - 1 = c$ or equivalently $\underline{(x+1)^2 + y^2 + z^2 = 1 + c}$

This is the sphere centered at $(-1, 0, 0)$ with radius $\sqrt{1+c}$.

For $c = 3, 8, 15$ we obtain sphere centered at $(-1, 0, 0)$ with

radii: $\sqrt{1+3} = 2$, $\sqrt{8+1} = 3$, $\sqrt{15+1} = 4$, respectively.

Example 2: Resolve into components the vectors $\vec{a}, \vec{c}, \vec{d}$ depicted below:

$\vec{a} = 3\vec{k}$, $\vec{c} = 3\vec{k}$

$\vec{d} = 2\vec{i} + 3\vec{k}$

Example 3: Let $\vec{v} = 3\vec{i} - 3\vec{j} + 8\vec{k}$. Find $||\vec{v}||$. Find the unit vector $\vec{u}$ in the direction of $\vec{v}$.

$||\vec{v}|| = \sqrt{9 + 9 + 64} = \underline{\sqrt{82}}$

$\vec{u} = \frac{1}{||\vec{v}||} \vec{v} = \frac{3}{\sqrt{82}}\vec{i} - \frac{3}{\sqrt{82}}\vec{j} + \frac{8}{\sqrt{82}}\vec{k}$