Class Worksheet 2/3/22 - Solutions

Example 1: Which of the tables of values below could represent a linear function? For those which could, find a formula for the function.

(A)

$x \setminus y$	0	3	6
0	1	-4	-9
2	4	-1	-6
4	7	2	-3
6	10	5	0

$\Delta y = 3$

Take $x=0$; that is, look at the first row. $\Delta y = 3$, $\Delta z = -4-1 = -5 = -9-(-4)$

The first row is linear with slope $\frac{\Delta z}{\Delta y} = -\frac{5}{3}$

So is every other row.

Each column is also linear with $\Delta x = 2$, $\Delta z = 3$, $\frac{\Delta z}{\Delta x} = \frac{3}{2}$. $f(x,y)$ is linear. $f(0,0)=1$. So $f(x,y) = \frac{3}{2}x - \frac{5}{3}y + 1$

(B)

$x \setminus y$	0	5	10
1	2	4	6
5	4	8	12
9	8	16	32

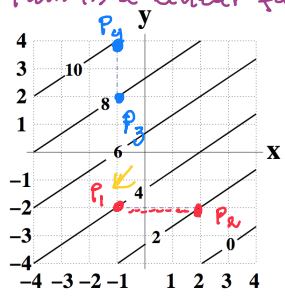
$\Delta y = 5$

The first row has slope $\frac{2}{5}$. The second row $\frac{4}{5}$. Not linear.

Example 2: Which of the following contour plots could represent a linear function? For those that could be linear, find a formula for the function.

This is a linear function.

(A)



Look at the points P_1 and P_2. $y=-2$ is fixed. $\Delta x = 2-(-1) = 3$, $\Delta z = 2-4 = -2$.

Hence, $m = -\frac{2}{3}$.

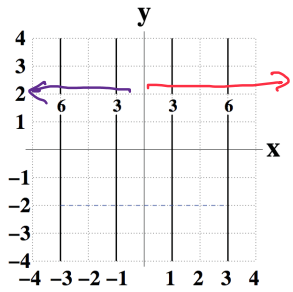
Look at P_3, P_4. x is fixed $x=-1$.

$\Delta y = 4-2 = 2$, $\Delta z = 10-8 = 2$.

So $m = \frac{2}{2} = 1$. A point $(x_0, y_0, z_0) = (-1,-2,4)$ is on the graph.

The formula for: $f(x,y) = 4 - \frac{2}{3}(x+1) + (y+2)$.

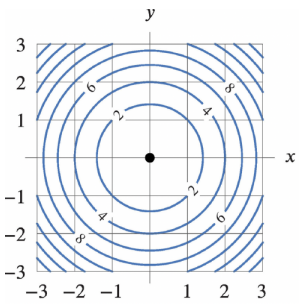
(B)



Values of z increase. In the opposite direction they increase as well.

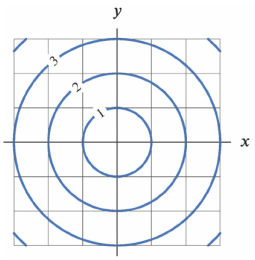
Not a linear function.

Example 3: Sketch a contour diagram of $z = \sqrt{x^2 + y^2}$. (Draw at least four marked contours.) How does the diagram differ from the contour diagram of the paraboloid $z = x^2 + y^2$?



$z = x^2 + y^2$

Both diagrams are in the notes for 12.3.



$z = \sqrt{x^2+y^2}$