Class Worksheet 2/24/22

Example 1:

Find the equation of the plane tangent to $z = 3e^y + x + x^4 + 6$ at the point $(1, 0, 11)$.

The equation of the plane is [ ]

Solution:
We have

$$z = 3e^y + x + x^4 + 6.$$

The partial derivatives are

$$\frac{\partial z}{\partial x}\bigg|_{(x,y)=(1,0)} = 4x^3 + 1\bigg|_{(x,y)=(1,0)} = 5$$

$$\frac{\partial z}{\partial y}\bigg|_{(x,y)=(1,0)} = 3e^y\bigg|_{(x,y)=(1,0)} = 3.$$

So the equation of the tangent plane is

$$z = 11 + 5(x - 1) + 3y = 6 + 5x + 3y.$$

Example 2:

For the differentiable function $h(x, y)$, we are told that $h(600, 100) = 300$ and $h_x(600, 100) = 13$ and $h_y(600, 100) = -9$. Estimate $h(605, 96)$.

$h(605, 96) \approx$ [ ]

Solution

The tangent plane approximation gives

$\Delta h \approx h_x \Delta x + h_y \Delta y,$

or

$h(x, y) \approx h(a, b) + h_x \Delta x + h_y \Delta y.$

With $(a, b) = (600, 100)$ and $(x, y) = (605, 96)$, we see that $\Delta x = 5$ and $\Delta y = -4$. Thus

$h(605, 96) \approx 300 + 13(5) - 9(-4) = 401.$

Using the information given, this is our best estimate for $h(605, 96)$. However, it may not be a good estimate if the derivatives are changing rapidly near the point $(600, 100)$.

Example 3:

Find the differential of the function $f(x, y) = 11 \sin(xy)$.

NOTE: Enclose arguments of functions in parentheses. For example, sin(2x).

$df =$ [ ] $dx +$ [ ] dy

Solution:

Since $f_x(x, y) = 11y\cos(xy)$ and $f_y(x, y) = 11x\cos(xy)$, we have:

$$\begin{aligned} df &= f_x(x, y)\,dx + f_y(x, y)\,dy \\ &= 11y\cos(xy)\,dx + 11x\cos(xy)\,dy \end{aligned}$$