Class Worksheet 10/05/21

Example 1: The temperature $H(x,t)$, in °C, in a room is a function of distance x, in meters, from a heater and time t, in minutes, after the heater has been turned on.

(a) What are the units of $H_x(x,t)$ and $H_t(x,t)$? What are the signs of the partial derivatives?

Units of $H_x(x,t)$ are $\frac{°C}{\text{meter}}$; units of $H_t(x,t)$ are $\frac{°C}{\text{min}}$.

Most likely $H_x(x,t) < 0$ as the farther we are from the heater, the cooler it is. $H_t(x,t) > 0$ as the more time elapses, the warmer it gets.

(b) Assume that $H(10,20) = 19$ and $H_x(10,20) = -0.5$. Estimate $H(11.5,20)$. Give units with your answer.

meters min °C −0.5 $\frac{°C}{\text{meter}}$ meters minutes.

10 meters from the heater, 20 min after the heater was turned on, the temperature is 19°C and the temperature is changing at the rate $-0.5\ \frac{°C}{\text{meter}}$. We estimate:

$$H(11.5, 20) \approx 19°C + (-0.5)\frac{°C}{m} \cdot 1.5m = 18.25°C$$

Example 2:

(a) Find the equation of the tangent plane to the graph of $z = f(x,y) = x^2 + y^2$ at the point $(a,b) = (3,4)$.

$f_x = 2x$, $f_x(3,4) = 6$, $f_y = 2y$, $f_y(3,4) = 8$, $f(3,4) = 25$

The tangent plane to the graph of $z = f(x,y)$ at $(3,4,25)$ is:

$$z = 25 + 6(x-3) + 8(y-4)$$

(b) Estimate $f(2.9, 4.2)$ using the local linearization at $(a,b) = (3,4)$. Compare your estimate to the actual value $f(2.9, 4.2)$.

$$f(a+\Delta x, b+\Delta y) \cong f(a,b) + f_x(a,b)\Delta x + f_y(a,b)\Delta y$$

$\Delta x = 2.9 - 3 = -0.1$, $\Delta y = 4.2 - 4 = 0.2$. Hence:

$$f(2.9, 4.2) = 25 + 6(-0.1) + 8 \cdot 0.2 = 26$$

The real value: $f(2.9, 4.2) = (2.9)^2 + (4.2)^2 = 26.05$

The approximation is relatively good as $(2.9, 4.2)$ is "close" to $(3,4)$.