Class Worksheet 2/17/22

Example 1:

(a) Let $f(x,y) = 3x^2y - 2x^3y^4$ find the partial derivative functions $f_x(x,y)$ and $f_y(x,y)$. Find $f_y(2,0)$.

$$f_x(x,y) = \frac{\partial}{\partial x}[3x^2y - 2x^3y^4] = 6xy - 6x^2y^4$$

$$f_y(x,y) = \frac{\partial}{\partial y}[3x^2y - 2x^3y^4] = 3x^2 - 8x^3y^3$$

$$f_y(2,0) = 3\cdot 2^2 - 8\cdot 2^3\cdot 0^3 = 12$$

(b) Let $f(x,y) = xe^{x^2y}$ find the partial derivative functions $f_x(x,y)$ and $f_y(x,y)$.

$$f_x = \frac{\partial}{\partial x}[xe^{x^2y}] = e^{x^2y} + x\frac{\partial}{\partial x}[e^{x^2y}] = e^{x^2y} + x\cdot 2xy\,e^{x^2y} = \underline{e^{x^2y} + 2x^2y\,e^{x^2y}}$$

Product rule

Chain rule

$$f_y = \frac{\partial}{\partial y}[xe^{x^2y}] = x\frac{\partial}{\partial y}[e^{x^2y}] = x\,e^{x^2y}\cdot x^2 = \underline{x^3e^{x^2y}}$$

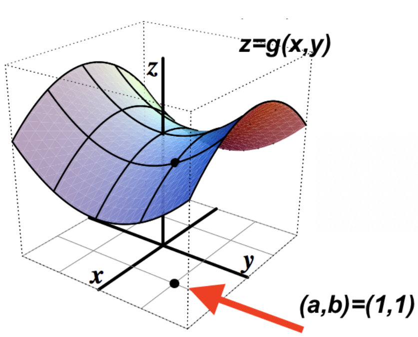
(c) Let $h(x,y) = \dfrac{x^2y}{x^3+y^2}$ find the partial derivative function $h_x(x,y)$.

$$h_x(x,y) = \frac{\partial}{\partial x}\left[\frac{x^2y}{x^3+y^2}\right] = \frac{\frac{\partial}{\partial x}[x^2y](x^3+y^2) - x^2y\frac{\partial}{\partial x}[x^3+y^2]}{(x^3+y^2)^2} =$$

quotient rule

$$= \frac{2xy(x^3+y^2) - x^2y\cdot 3x^2}{(x^3+y^2)^2} = \frac{2xy(x^3+y^2) - 3x^4y}{(x^3+y^2)^2}$$

Example 2: The graph of a function $z = g(x,y)$ is shown below. Is $g_x(1,1)$ positive or negative? Is $g_y(1,1)$ positive or negative? Explain your answers.



$g_x(1,1) = \frac{d}{dx}\Big|_{x=1}[g(x,1)] < 0$ as the cross-section $g(x,1)$ is decreasing at $x=1$.

$g_y(1,1) = \frac{d}{dy}\Big|_{y=1}[g(1,y)] > 0$ as the cross-section $g(1,y)$ is increasing at $y=1$.