Class Worksheet 2/15/2022

Example 1: Let $\vec{v} = \vec{i} + 2\vec{j}$ and $\vec{w} = -\vec{i} + 2\vec{j} + \vec{k}$. Find $\vec{v} \times \vec{w}$ and $\vec{w} \times \vec{v}$.

$$\vec{v} \times \vec{w} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 0 \\ -1 & 2 & 1 \end{vmatrix} = \vec{i} \begin{vmatrix} 2 & 0 \\ 2 & 1 \end{vmatrix} - \vec{j} \begin{vmatrix} 1 & 0 \\ -1 & 1 \end{vmatrix} + \vec{k} \begin{vmatrix} 1 & 2 \\ -1 & 2 \end{vmatrix} =$$

$$= \vec{i}\,(2\cdot 1 - 2\cdot 0) - \vec{j}\,(1\cdot 1 - (-1)\cdot 0) + \vec{k}\,(1\cdot 2 + 1\cdot 2) =$$

$$= 2\vec{i} - \vec{j} + 4\vec{k}$$

$$\boxed{\vec{v} \times \vec{w} = 2\vec{i} - \vec{j} + 4\vec{k}}$$

$$\vec{w} \times \vec{v} = -(\vec{v} \times \vec{w}) = -(2\vec{i} - \vec{j} + 4\vec{k}) = \boxed{-2\vec{i} + \vec{j} - 4\vec{k}}$$

Example 2:

(a) Find the equation of the plane, L, that contains the points $P_1 = (1, 2, -1)$, $P_2 = (2, 3, 0)$, $P_3 = (3, -1, 2)$.

(b) Find a unit normal vector to the plane L.

(a) The vectors $\overrightarrow{P_1P_2}$ and $\overrightarrow{P_1P_3}$ are on L. Hence, the vector $\vec{n} = \overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3}$ which is perpendicular to both $\overrightarrow{P_1P_2}$, $\overrightarrow{P_1P_3}$ is normal to L. We calculate $\vec{n}$.

$$\overrightarrow{P_1P_2} = \vec{i} + \vec{j} + \vec{k}, \quad \overrightarrow{P_1P_3} = 2\vec{i} - 3\vec{j} + 3\vec{k}$$

$$\vec{n} = \overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ 2 & -3 & 3 \end{vmatrix} = \vec{i}\cdot 6 - \vec{j} - 5\vec{k} = 6\vec{i} - \vec{j} - 5\vec{k}$$

P_1 is on L so the equation of L is:

$$\boxed{6(x-1) - (y-2) - 5(z+1) = 0}$$

(b) $\frac{1}{\|\vec{n}\|}\vec{n}$ is a unit normal vector $\vec{u}$ to L. $\|\vec{n}\| = \sqrt{36 + 1 + 25} = \sqrt{62}$.

$$\boxed{\vec{u} = \frac{6}{\sqrt{62}}\vec{i} - \frac{1}{\sqrt{62}}\vec{j} - \frac{5}{\sqrt{62}}k}$$