Class Worksheet 2/10/2022

Example 1: Let $\vec{v} = 3\vec{i} + 3\vec{j} + 4\vec{k}$ and $\vec{w} = 3\vec{i} + 4\vec{k}$. Find an angle, in degrees, between the two vectors.

$\vec{v} \cdot \vec{w} = \|\vec{v}\| \, \|\vec{w}\| \cos\theta$ from the definition of the dot product.

We have:

$\vec{v} \cdot \vec{w} = 3 \cdot 3 + 3 \cdot 0 + 4 \cdot 4 = 25$, $\|\vec{v}\| = \sqrt{9+9+16} = \sqrt{34}$, $\|\vec{w}\| = \sqrt{9+16} = \sqrt{25} = 5$.

So: $25 = \sqrt{34} \cdot 5 \cos\theta$, $\cos\theta = \dfrac{25}{5\sqrt{34}}$, $\boxed{\theta = \cos^{-1}\left(\dfrac{5}{\sqrt{34}}\right) = 30.96^{\circ}}$

Example 2: Which of the following are equations of planes? For those that are, find a normal vector for the plane.

(a) $2x - 3y - 5z = 4$ (b) $z = 4 - 2y - 4x$ (c) $2x^2 + 3(y-2)^2 - z = 0$

The equation (a) represents a plane. A normal vector: $\vec{n} = 2\vec{i} - 3\vec{j} - 5\vec{k}$

The equation (b) represents a plane. We have to rewrite it in the form $ax + by + cz = d$: $4x + 2y + z = 4$. A normal vector $\vec{n} = 4\vec{i} + 2\vec{j} + \vec{k}$.

(c) is a quadratic in x and y so it does not represent a plane.

Example 3: Find an equation of the plane, L, parallel to the plane $x + 2y - 3z = 1$ and passing through the point $(-1, 2, 0)$.

A normal vector to the plane $x + 2y - 3z = 1$ is $\vec{n} = \vec{i} + 2\vec{j} - 3\vec{k}$.

Parallel planes have the same normal vectors so $\vec{n}$ is normal to L. We have a normal vector and a point $(x_0, y_0, z_0) = (-1, 2, 0)$ on L. Hence:

The equation of L is: $\boxed{(x+1) + 2(y-2) - 3z = 0}$