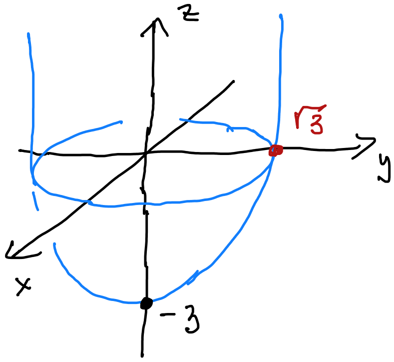
Class Worksheet 1/27/22 - Solutions

Example 1: Sketch by hand the graph of $z = h(x,y) = x^2 + y^2 - 3$. Sketch and describe the intersection of the graph with the xy-plane.

$z = x^2 + y^2 - 3$ is the standard paraboloid shifted 3 units down.

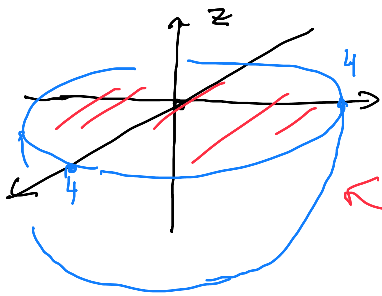


The intersection of the shifted paraboloid with the xy-plane, i.e, $z=0$ plane, is the curve:

$$0 = x^2+y^2-3 \quad , \quad x^2+y^2 = 3,$$

which is the circle on the xy-plane centered at the origin with radius $\sqrt{3}$.

Example 2: The graph of $z = f(x,y)$ is the lower hemisphere of the sphere centered at the origin with radius 4. Find a formula for $f(x,y)$. What is the domain of f?



The equation of the sphere centered at the origin with radius 4 is:

$$x^2+y^2+z^2 = 16$$

The lower hemisphere is:

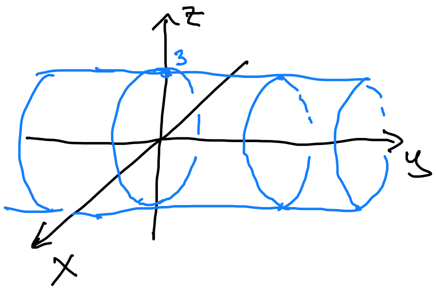
$$z = -\sqrt{16-x^2-y^2}$$

Hence, the lower hemisphere is the graph of

$$f(x,y) = -\sqrt{16-x^2-y^2}.$$

$f(x,y)$ is defined for pairs (x,y) such that $x^2+y^2 \leq 16$. Hence, the domain of f is the disc on the xy-plane centered at the origin with radius 4.

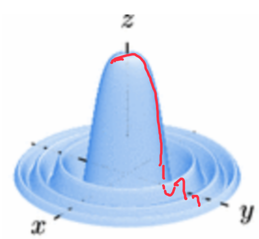
Example 3: Find an equation of the cylinder about the y-axis with radius 3.



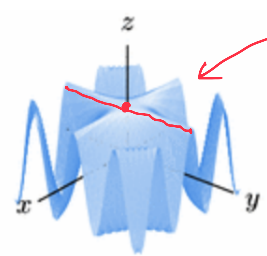
The equation of the cylinder is

$$x^2+z^2=9$$

Example 4: Which of the following surfaces is the graph of the function $z = \cos(xy)$?



(A)



(B)

Take $f(x,y)=\cos(xy)$. Look at the cross-section of $z=f(x,y)$ with the plane $x=0$:

$f(0,y)=\cos(0\cdot y)=\cos(0)=1$.

The cross-section is the line $z=f(0,y)=1$ in the yz-plane.

This is true for the graph (B) but not for the graph (A).