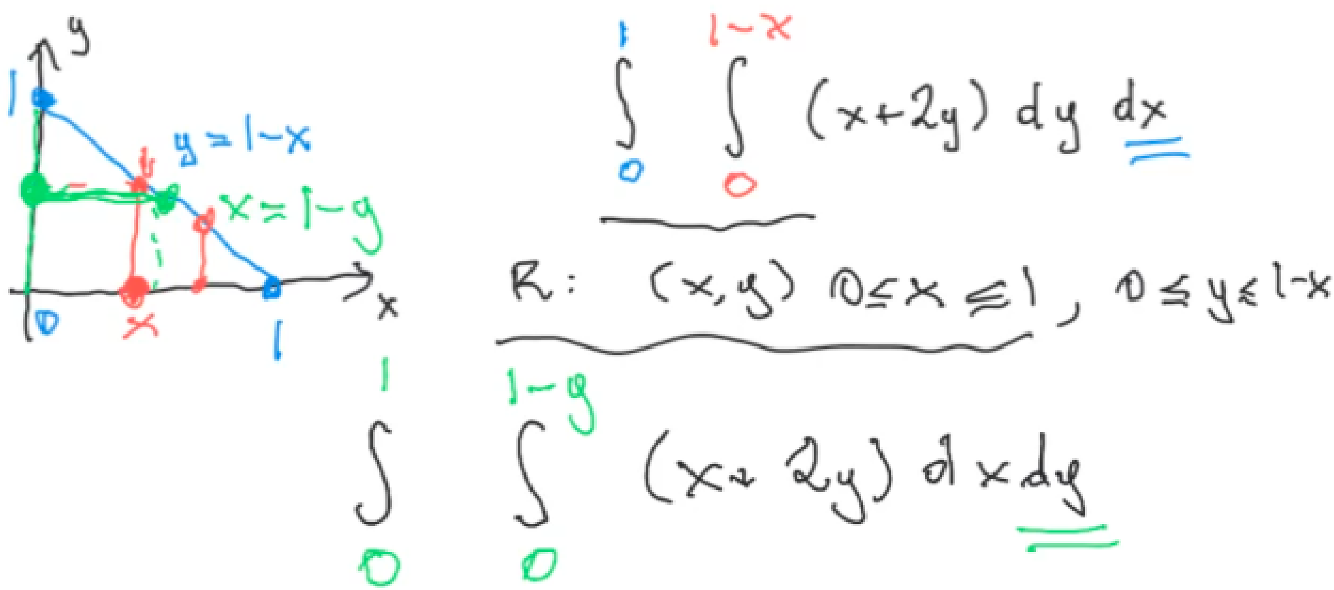
Example 1: Find

$$\int_R (x+2y)dA$$

where R is the triangle with vertices $(0,0),(1,0),(0,1)$.



The picture above is a screen shot from the video linked on our Brightspace page. Please see the video for explanations.

Note: In the example presented here, the region R is symmetric with respect to x and y. Hence, so are the limits in the iterated integrals. In general, it may not be the case.

Below, we evaluate both iterated integrals.

$$\int_0^1 \left(\int_0^{1-x}(x+2y)dy\right)dx = \int_0^1 \left[(xy+y^2)\Big|_{y=0}^{y=1-x}\right]dx = \int_0^1 (x(1-x)+(1-x)^2)dx =$$

$$= \int_0^1 (1-x)(x+1-x)dx = \int_0^1 (1-x)dx = \frac{1}{2}$$

We don't have to evaluate the second iterated integral as by the theorem about iterated integrals the values of both integrals are the same but for practice sake let's evaluate the second iterated integral:

$$\int_0^1 \left(\int_0^{1-y} (x+2y)dx \right) dy = \int_0^1 \left[(\frac{1}{2}x^2 + 2yx) \Big|_{x=0}^{x=1-y} \right] dy =$$

$$= \int_0^1 (\frac{1}{2}(1-y)^2 + 2(1-y)y)dy = \frac{1}{2}$$