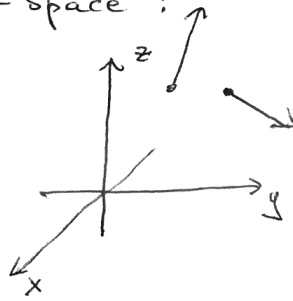
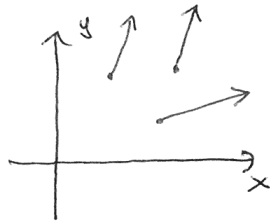


## 13.1-13.2 Review and Examples

①

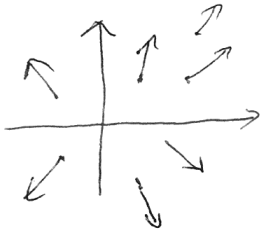
Vectors are objects that have magnitude as well as direction. Geometrically vectors are usually represented by arrows on the  $xy$ -plane or in the  $xyz$ -space;



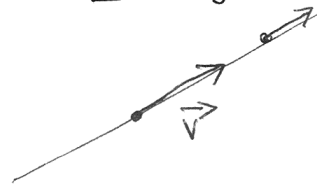
Two vectors that have the same magnitude and direction are considered the same - the tail of a vector does not matter.

Many objects in applications are vectors.

### Force Field



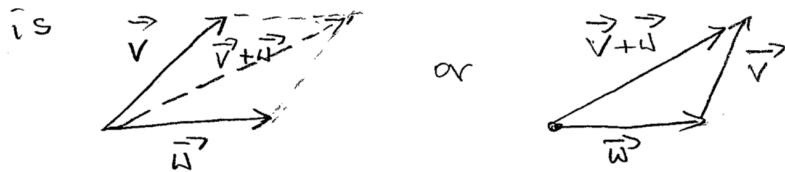
### Velocity



All operations and properties of vectors are usually defined in two ways: purely geometrically - in terms of arrows - and algebraically - in terms of components. What are components of a vector? We will discuss it soon.

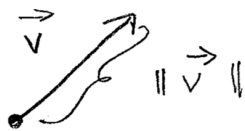
When you talk about vectors numerical quantities that do not have direction are usually called scalars.

We define operations on vectors:  $\vec{v} + \vec{w}$ ,  $\|\vec{v}\|$ ,  $\alpha\vec{v}$ ,  $\vec{v} - \vec{w}$  in purely geometric terms as follows. Let two vectors  $\vec{v}$ ,  $\vec{w}$  and a scalar  $\alpha$  be given. Then the sum  $\vec{v} + \vec{w}$

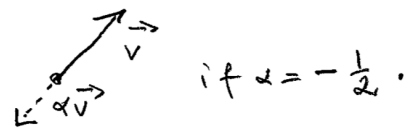
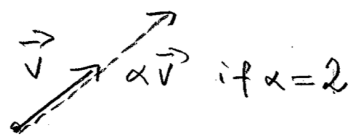


(Remember the tail of a vector doesn't matter; we can move it freely.)

The magnitude  $\|\vec{v}\|$  is the length of the arrow representing  $\vec{v}$ . In other words, the distance between the tail and the endpoint of  $\vec{v}$ :

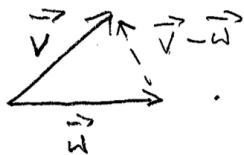


$\alpha\vec{v}$  is a vector parallel to  $\vec{v}$  with its length rescaled by the factor  $\alpha$ . If  $\alpha > 0$ ,  $\alpha\vec{v}$  points in the same direction as  $\vec{v}$ , if  $\alpha < 0$ ,  $\alpha\vec{v}$  points in the opposite direction.



$$\vec{v} - \vec{w} = \vec{v} + (-\vec{w}), \text{ where } -\vec{w} = -1 \cdot \vec{w}.$$

Geometrically:



Ex 1:

The vectors  $\vec{w}$  and  $\vec{u}$  are in Figure 12.16. Match the vectors  $\vec{p}, \vec{q}, \vec{r}, \vec{s}, \vec{t}$  with five of the following vectors:  $\vec{u} + \vec{w}, \vec{u} - \vec{w}, \vec{w} - \vec{u}, 2\vec{w} - \vec{u}, \vec{u} - 2\vec{w}, 2\vec{w}, -2\vec{w}, 2\vec{u}, -2\vec{u}, -\vec{w}, -\vec{u}$ .

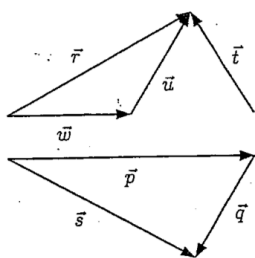
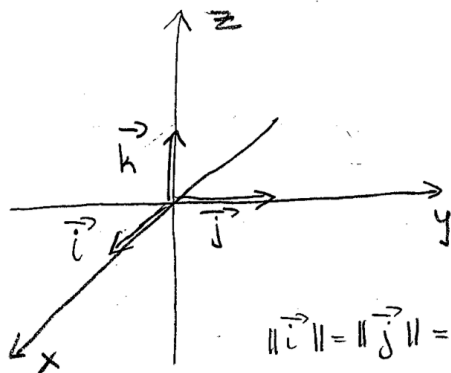


Figure 12.16

$$\begin{aligned} \vec{r} &= \vec{w} + \vec{u}, & \vec{t} &= \vec{u} - \vec{w} \\ \vec{p} &= 2\vec{w}, & \vec{q} &= -\vec{u} \\ \vec{s} &= \vec{p} + \vec{q} = 2\vec{w} - \vec{u} \end{aligned}$$

We choose basic vectors  $\vec{i}, \vec{j}, \vec{k}$  in the  $xyz$ -space as follows:



$$\|\vec{i}\| = \|\vec{j}\| = \|\vec{k}\| = 1$$

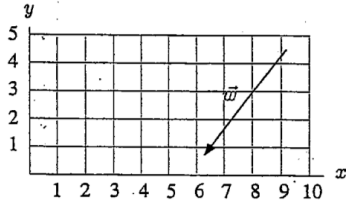
Every vector  $\vec{v}$  in the  $xyz$ -space can be uniquely represented as:

$$\vec{v} = v_1 \vec{i} + v_2 \vec{j} + v_3 \vec{k}.$$

The numbers  $v_1, v_2, v_3$  are called components of  $\vec{v}$ .

Ex 2: Resolve the vector  $\vec{w}$  below into its  $\vec{i}, \vec{j}$  components:  $\vec{w} = w_1 \vec{i} + w_2 \vec{j}$ ;

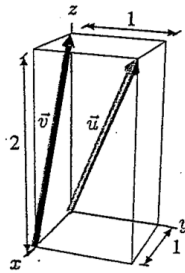
(3)



$\vec{w}$  displaces its tail about 2.5 units in the negative  $\vec{i}$  direction and about 3.5 units in the negative  $\vec{j}$  direction.

Hence:  $\vec{w} = -2.5 \vec{i} - 3.5 \vec{j}$

Ex 3: Resolve  $\vec{u}$  and  $\vec{v}$  into their  $\vec{i}, \vec{j}, \vec{k}$  components:



Looking at how many units  $\vec{u}$  displaces in the  $\vec{i}, \vec{j}, \vec{k}$  directions, we see that:

$$\vec{u} = \vec{i} + \vec{j} + 2\vec{k}$$

Similarly,

$$\vec{v} = -\vec{i} + 2\vec{k}$$

All operations can easily be expressed in terms of components.

Let  $\vec{v} = v_1 \vec{i} + v_2 \vec{j} + v_3 \vec{k}$ ,  $\vec{w} = w_1 \vec{i} + w_2 \vec{j} + w_3 \vec{k}$  be given. Then:

$$\vec{v} + \vec{w} = (v_1 + w_1) \vec{i} + (v_2 + w_2) \vec{j} + (v_3 + w_3) \vec{k},$$

$$\vec{v} - \vec{w} = (v_1 - w_1) \vec{i} + (v_2 - w_2) \vec{j} + (v_3 - w_3) \vec{k},$$

$$\alpha \vec{v} = (\alpha v_1) \vec{i} + (\alpha v_2) \vec{j} + (\alpha v_3) \vec{k} \text{ for any scalar } \alpha,$$

$$\|\vec{v}\| = \sqrt{v_1^2 + v_2^2 + v_3^2}.$$

We define the zero vector  $\vec{0}$  as:

$$\vec{0} = 0\vec{i} + 0\vec{j} + 0\vec{k}.$$

A vector  $\vec{u}$  is called a unit vector if  $\|\vec{u}\| = 1$ .

Observe the following:

$$\|\alpha\vec{v}\| = |\alpha| \|\vec{v}\|$$

for every vector  $\vec{v} = v_1\vec{i} + v_2\vec{j} + v_3\vec{k}$  and scalar  $\alpha$ . Of course:

$$\begin{aligned} \|\alpha\vec{v}\| &= \|\alpha v_1\vec{i} + \alpha v_2\vec{j} + \alpha v_3\vec{k}\| = \sqrt{\alpha^2 v_1^2 + \alpha^2 v_2^2 + \alpha^2 v_3^2} = \\ &= \sqrt{\alpha^2} \sqrt{v_1^2 + v_2^2 + v_3^2} = |\alpha| \|\vec{v}\|. \end{aligned}$$

Ex 4: Let  $\vec{v} = \vec{i} - \vec{j} + 2\vec{k}$ . Find the unit vector  $\vec{u}$  in the direction of  $\vec{v}$ .

Note the  $\frac{1}{\|\vec{v}\|} \vec{v}$  points in the direction of  $\vec{v}$ . Also,

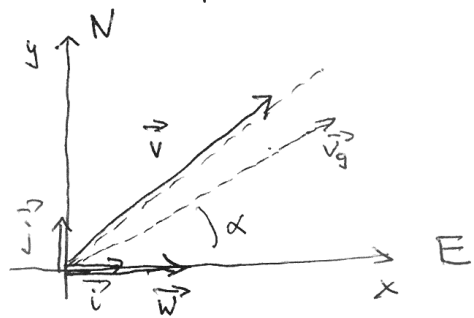
$$\left\| \frac{1}{\|\vec{v}\|} \vec{v} \right\| = \frac{1}{\|\vec{v}\|} \|\vec{v}\| = 1. \quad \text{So we can take}$$

$$\vec{u} = \frac{\vec{v}}{\|\vec{v}\|}.$$

$$\|\vec{v}\| = \sqrt{1^2 + 1^2 + 2^2} = \sqrt{6}. \quad \text{Hence } \vec{u} = \frac{1}{\sqrt{6}} \vec{i} - \frac{1}{\sqrt{6}} \vec{j} + \frac{2}{\sqrt{6}} \vec{k}.$$

5

Ex 5: An airplane is heading northeast at an airspeed of 700 km/hr, but there is a wind blowing from the west at 60 km/hr. In what direction does the plane end up flying? What is its speed and velocity relative to the ground?



$$\vec{v} = \frac{700}{\sqrt{2}} \vec{i} + \frac{700}{\sqrt{2}} \vec{j}$$
$$\vec{w} = 60 \vec{i}$$

$$\vec{v}_g = \left( \frac{700}{\sqrt{2}} + 60 \right) \vec{i} + \frac{700}{\sqrt{2}} \vec{j} \approx 555 \vec{i} + 495 \vec{j}$$

Ground speed =  $\|\vec{v}_g\| = \sqrt{555^2 + 495^2} \approx 744$  km/hr.

$$\alpha = \arctan\left(\frac{495}{555}\right) \approx 42^\circ.$$