Trigonometric Review

The Unit Circle

This circle is a circle with radius = 1, hence the name, the unit circle.

The equation for the circle is $x^2 + y^2 = 1$

The circumference of the circle is $2\pi r$, so the circumference of the unit circle is $2\pi$.

¼ of the way around is $\pi/2$
½ of the way around is $\pi$
¾ of the way around is $3\pi/4$
all the way around is $2\pi$

Note that on the unit circle the $x$-values represent the cosine value and the $y$-values represent the sine value, so if $x^2 + y^2 = 1$, then $(\cos \theta)^2 + (\sin \theta)^2 = 1$.

1. Fill in the table below:

<table>
<thead>
<tr>
<th>$\theta$ (Radians)</th>
<th>$\cos \theta$</th>
<th>$\sin \theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$\pi/2$</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$\pi$</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>$3\pi/2$</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>$2\pi$</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

2. What is: $\cos\left(\frac{\pi}{4}\right) \quad \frac{\sqrt{2}}{2} \quad \sin\left(\frac{\pi}{4}\right) \quad \frac{\sqrt{2}}{2}$
Trigonometric Graphs

1. Graph $\cos t$ and $\sin t$.

![Graph of $\cos t$ and $\sin t$]

2. Match four of the following functions to the graphs below; then graph the remaining two functions.

   a. $f(x) = 1 + \sin x$
   b. $g(x) = 1 - \sin x$
   c. $h(x) = 3\cos x$
   d. $r(x) = \cos 2x$
   e. $s(x) = 3\sin x$
   f. $m(x) = \sin 2x$

   ![Graphs of the functions]

   1. $f(x)$
   2. $d(x)$
   3. $e(x)$
   4. $b(x)$
   5. $a(x) = 1 + \sin x$
   6. $h(x) = 3\cos x$
Radians and Degrees

Conversions: \( \pi \) radians = 180 degrees  
1 radian = \( \frac{180}{\pi} \) degrees  
1 degree = \( \frac{\pi}{180} \) radians

1. Find the radian measure of the angle when given the degree measure:
   a. 36 degrees = \( \frac{\pi}{5} \) radians  
b. 200 degrees = \( \frac{10\pi}{9} \) radians  
c. 45 degrees = \( \frac{\pi}{4} \) radians  
d. -72 degrees = \( -\frac{2\pi}{5} \) radians  
e. 60 degrees = \( \frac{\pi}{3} \) radians  
f. 115 degrees = \( \frac{31\pi}{15} \) radians  
g. -135 degrees = \( -\frac{3\pi}{4} \) radians  
h. 150 degrees = \( \frac{5\pi}{6} \) radians  
i. -420 degrees = \( -\frac{7\pi}{3} \) radians

2. Find the degree measure of the angle with the following radian measure:
   a. \( \frac{3\pi}{4} \) = 135 degrees  
b. \( -\frac{7\pi}{2} \) = -450 degrees  
c. \( \frac{5\pi}{6} \) = 150 degrees  
d. \( -\frac{\pi}{12} \) = -15 degrees  
e. -1.5 = \( -\frac{270}{\pi} \) degrees  
f. \( \frac{2\pi}{9} \) = 40 degrees  
g. \( \frac{\pi}{5} \) = 36 degrees  
h. \( \frac{\pi}{18} \) = 10 degrees  
i. \( \frac{5\pi}{3} \) = 300 degrees

Trigonometric Identities

Simplify the following trigonometric expressions:

1. \( (\sin \theta)^2 + (\cos \theta)^2 - 1 = 0 \)
2. \( (\sin \theta + \cos \theta)^2 + 2 \cos \theta = 1 + 2 \cos \theta + 2 \sin \theta \cos \theta \)
3. \( (\sin \theta)(\cos \theta)^2 + (\sin \theta)^3 - 2 = \sin \theta - 2 \)
4. \( 2(\cos \theta)^2 + 2(\sin \theta)^2 + 1 = 3 \)