Mathematica Project 1
25 points

Many, many years ago there was a commerical for Coors beer where the quarterback of the Denver Broncos, John Elway, threw a baseball from homeplate over the left field fence of Coors field. Is this possible? If it is possible what angle and how fast did he have to throw the ball?

We will ignore air friction (i.e. drag on the ball), mass/size of the ball, and density of the air. We will use a simplified equation to analyze the path of a ball being thrown. The position \((x, f[x])\) of a ball in the \(xy\)-plane is given as

\[
f[x] = \tan[a]x - \frac{16}{v^2}(1 + \tan^2[a])x^2
\]

The \(x\) value is distance from homeplate, \(y = f[x]\) is the height the ball is from the ground, \(a\) is the angle the ball is thrown, and \(v\) is the velocity (in \(\text{ft/sec}\)) the ball is thrown.

You may work in groups up to 3 students. The students in your group must be from your class section. There will be a Mathematica skills exam on the commands used in the projects.

1. **Text mode**
   Start a new Mathematica notebook. Using text mode, type your name (include all names of students in your group, maximum of 3 students per group) section number, name of your instructor, due date, and Project 1.

   Given \(f[x] = \tan[a]x - \frac{16}{v^2}(1 + \tan^2[a])x^2\)

2. **Command mode (4 points)**
   Using the Mathematica command for a function \(f[x]\) input the given function. \textit{Hint: If you use other Mathematica functions, such as Plot or Solve use }\textit{f[x]}\textit{ to refer back to your input function.}
3. **Plot (4 points)**

Set the angle \( a = \frac{\pi}{6} \) (\( \pi \) in Mathematica is \( \text{Pi} \)) and initial velocity \( v = 100 \text{ ft/sec} \) and using the Mathematica command `Plot` with \( x \) between 0 and 350 create a plot of \( f[x] \). Pick other values for \( a \) and \( v \) and plot \( f[x] \) for \( x \) between 0 and 350. For full credit you must pick at least two other choices for \( a \) and \( v \). Notice: No two student groups should have the same plots and choices for \( a \) and \( v \).

4. **Command mode and Plot (4 points)**

Using Mathematica command `Clear`, clear the variables \( a \) and \( v \). Now use the Mathematica command `Solve` to find when \( f[x] = 0 \). The solution to \( f[x] = 0 \) does depend on \( v \) and \( a \), i.e. \( x \) is a function of \( v \) and \( a \). For three different values of \( v \) (you pick the different values of \( v \)), plot \( x[a] \) for your three different choices on one graph. **Determine the value of \( a \) that yields the largest \( x \) value.** Notice: No two student groups should have the same plots and choices for \( v \).

5. **Command mode and Text mode (4 points)**

When \( x = 347 \), we need to have \( y = f[347] > 2 \) in order for the baseball to go over the left field fence. Use the value of the angle \( a \) found in problem 4 that yielded the largest \( x \) value and the Mathematica command `Solve` to find when \( f[347] = 2 \). **What is the value of \( v \)?** Using the fact that 1 foot per second = 0.682 mph translate your answer into mph. Is it humanly possible for John Elway to throw a ball over the left field fence?

6. **Command mode and Plot (4 points)**

Setting \( x = 347 \) and \( y = f[347] = 2 \) then we have

\[
2 = \text{Tan}[a]347 - \frac{16}{v^2}(1 + \text{Tan}^2[a])(347)^2.
\]

Use Mathematica and the equation above to obtain an equation for \( v \) in terms of \( a \). (\( \text{Hint: You are solving for } v \).) You will end up with two equations for \( v \) that only differ by sign. Set \( v \) to be the positive equation, (highlighting output and copying and pasting works in Mathematica). Differentiate \( v \) with respect to \( a \) (\( \text{Hint: you can use the Mathematica command } \prime, \text{ i.e. } v'[a] \text{ works as the derivative} \)) and plot the derivative \( v'[a] \) and the function \( v[a] \) on the same graph between the \( a \) values \( \frac{\pi}{10} \) and \( \frac{2\pi}{3} \).

7. **Text mode (5 points)**

Analyze the graph from problem 6. Where does the derivative cross the \( a \)-axis, i.e. where is \( v'[a] = 0 \)? What does this mean? Use Textmode in Mathematica to answer the questions. **Referring to Section 2.4 Interpretations of the Derivative in your Calculus book, give the practical meaning of the derivative \( v'[a] = \frac{dv}{da} \) in reference to a change in the angle \( a \) and the initial velocity \( v \) of the ball?** (\( \text{Hint: Examine what happens around the optimal angle } a \text{ and what happens when } a \text{ gets close to } \pi/2. \))