1. Write an integral which gives the exact length of the curve \( f(x) = \cos x \) from \( x = 0 \) to \( x = \pi \).

\[ \text{Sol:} \quad \int_0^\pi \sqrt{1 + (- \sin x)^2} \, dx \]

2. Find the volume of the solid obtained by rotating the region bounded by \( y = x^3 \), \( x = 1 \), and \( y = 0 \) around the \( x \)-axis.

\[ \text{Sol:} \quad \int_0^1 \pi (x^3)^2 \, dx = \frac{\pi}{7} \]

3. A triangle (in the \( xy \)-plane) has its vertices at the points \((-1, 0)\), \((0, 1)\), and \((1, 0)\). Find the total mass of the region enclosed by this triangle if its density function is given by \( \delta(x) = 1 + x^2 \) grams per cm\(^2\).

\[ \text{Sol:} \quad \text{The 2 lines representing the sides of the triangle that are not on the } x \text{-axis are } y = 1 + x \text{ and } y = 1 - x. \] So the total mass can be expressed as the sum of two integrals

\[ \int_{-1}^0 (1 + x^2)(1 + x) \, dx + \int_0^1 (1 + x^2)(1 - x) \, dx = 1.16 \]

4. A water tank has the shape of a right circular cylinder, with height 30 ft and radius 8 ft. If the tank is half full of water, find the work required to take the water out of the tank from the top. The density of water is 62.4 lbs per ft\(^3\).

\[ \text{Sol:} \quad 62.4 \int_{15}^{30} \pi 8^2 h \, dh = 4,234,364 \]

5. A dam has the shape of a trapezoid with horizontal parallel sides measuring 30 ft. (bottom) and 40 ft (top). The height of the dam is 30 ft., and one vertical side is perpendicular to both base and top. The dam has water up to the top. Find the total force of the water on the dam. (Water weighs 62.4 lbs/ft\(^3\).)

\[ \text{Sol:} \quad 62.4 \int_0^{30} h (30 + (30 - h)/3) \, dh = 936,000 \]

6. Decide if each of the following infinite series converges or diverges. Explain your answer using theorems from section 9.1 and 9.2.
(a) \[
\sum_{n=0}^{\infty} \frac{n^2}{n^3 - n}
\]

(b) \[
\sum_{n=1}^{\infty} 1 - \frac{1}{n^2}
\]

(c) \[
\sum_{n=3}^{\infty} \frac{(-1)^{n+1}}{5^n}
\]

Sol:

(a) Use the integral test. Compare to \( \int_{1}^{\infty} \frac{x^2}{x^3 - 1} \, dx \) which when reduced is \( \int_{1}^{\infty} \frac{1}{x} \, dx \) which diverges. Therefore, the series diverges.

(b) \( \lim_{n \to \infty} 1 - \frac{1}{n^2} = 1 \) Since the \( n^{th} \) term does not go to zero, the series diverges.

(c) This is a geometric series with \( a = -1 \) and \( x = -1/5 \), which converges.

7. Suppose \( F(x) \) is the cumulative distribution function for heights (in meters) for trees in a forest.

(a) Explain in terms of trees the meaning of the statement \( F(7) = .6 \).

(b) Which is greater, \( F(7) \) or \( F(6) \)? Justify your answer in terms of trees.

Sol:

(a) The probability that a tree chosen at random has height at most 7m is .6.

(b) \( F(7) \geq F(6) \) since the probability that the height is at most 7m must be at least as big as the prob the height is at most 6m.

8. A ball is dropped from the height of 15 ft. Each time it hits the ground, it rebounds to 4/5 of the height it attained on the previous bounce. Assuming that this process goes on forever, find the total distance the ball travels.

Sol:

\[
15 + 2 \cdot 15 \cdot \frac{4}{5} + 2 \cdot 15 \cdot \left( \frac{4}{5} \right)^2 + 2 \cdot 15 \cdot \left( \frac{4}{5} \right)^3 + \cdots
\]

\[
= \sum_{k=0}^{\infty} 30 \left( \frac{4}{5} \right)^k - 15 = \frac{30}{1 - 4/5} - 15 = 135
\]

9. Consider the function \( p(x) \) defined by \( p(x) = x^2/9 \) for \( p < x < 3 \) and \( p(x) = 0 \) otherwise.
(a) Show that \( p(x) \) is a probability density function.
(b) What proportion of \( x' \)s have values between 1.2 and 1.8.
(c) Compute the mean of this distribution.

**Sol:**

(a) \( p(x) \geq 0 \) for all \( x \) and
\[
\int_0^3 \frac{x^2}{9} \, dx = 1
\]

(b)
\[
\int_{1.2}^{1.8} \frac{x^2}{9} \, dx = .152
\]

(c)
\[
\int_0^x \frac{x^3}{9} \, dx = 2.25
\]

10. Find the sum of the geometric series
\[
\sum_{n=2}^{\infty} \frac{2^n + (-1)^n}{6^n}
\]

**Sol:**
\[
\sum_{n=2}^{\infty} \left( \frac{2}{6} \right)^n + \sum_{n=2}^{\infty} \left( \frac{-1}{6} \right)^n = \sum_{n=0}^{\infty} \frac{1}{9} \left( \frac{2}{6} \right)^n + \sum_{n=0}^{\infty} \frac{1}{36} \left( \frac{-1}{6} \right)^n = .19
\]