1. Find the volume of the solid obtained by rotating the region bounded by \( y = x^3, \ x = 1, \) and \( y = 0 \) around the \( x \)-axis.

**Sol:**

\[
\int_0^1 \pi (x^3)^2 \, dx = \frac{\pi}{7}
\]

2. Find the length of the parametric curve given by \( x = \cos(e^t), \ y = \sin(e^t), \ 0 \leq t \leq 1. \)

**Sol:**

\[
\begin{align*}
\int_0^1 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \, dt &= \int_0^1 \sqrt{(-e^t \sin(e^t))^2 + (e^t \cos(e^t))^2} \, dt \\
&= \int_0^1 \sqrt{e^{2t} \sin^2(e^t) + \cos^2(e^t)} \, dt \Rightarrow \int_0^1 e^t \, dt = e - 1
\end{align*}
\]

3. A triangle (in the \( xy \)-plane) has its vertices at the points \((-1,0), \ (0,1), \) and \((1,0). \) Find the total mass of the region enclosed by this triangle if its density function is given by \( \delta(x) = 1 + x^2 \) grams per cm\(^2\).

**Sol:** The 2 lines representing the sides of the triangle that are not on the \( x \)-axis are \( y = 1+x \) and \( y = 1-x. \) So the total mass can be expressed as the sum of two integrals

\[
\int_{-1}^0 (1+x^2)(1+x) \, dx + \int_0^1 (1+x^2)(1-x) \, dx = 1.16
\]

4. A water tank has the shape of a right circular cylinder, with height 30 ft and radius 8 ft. If the tank is half full of water, find the work required to take the water out of the tank from the top. The density of water is 62.4 lbs per ft\(^3\).

**Sol:**

\[
62.4 \int_{15}^{30} \pi 8^2 \ h \, dh = 4,234,364
\]

5. A person deposits money into a savings account which pays 5 per cent interest compounded continuously, at a rate of 1500 dollars per year for 25 years.

(a) Calculate the balance in the account at the end of 25 years.

(b) Calculate the amount of money actually deposited into the account.

(c) Calculate the interest earned during the 25 years.

**Sol:**
(a) \[ \int_{0}^{25} 1500 e^{0.05(25-t)} \, dt = 74,710.28 \]

(b) 37,500

(c) 37210.28

6. Use the integral test to determine the convergence or divergence of the series.

\[ \sum_{n=0}^{\infty} \frac{1}{n(\ln n)^2} \]

**Sol:** Use the integral test. Compare to \( \int_{1}^{\infty} \frac{1}{x \ln^2 x} \, dx \) which we see converges by substituting \( u = \ln x \) so \( du = 1/x \, dx \). We have

\[ \int_{2}^{b} \frac{1}{x \ln^2 x} \, dx = \int_{2}^{b} \frac{1}{u^2} \, du = -\frac{1}{u} \bigg|_{2}^{b} = -\frac{1}{\ln x} \bigg|_{2}^{b} = \frac{1}{\ln 2} - \frac{1}{\ln b}. \]

\[ \lim_{b \to \infty} \left( \frac{1}{\ln 2} - \frac{1}{\ln b} \right) = \frac{1}{2} \]

We conclude that \( \int_{1}^{\infty} \frac{1}{x \ln^2 x} \, dx \) converges. So by the integral test, \( \sum_{n=0}^{\infty} \frac{1}{n(\ln n)^2} \) converges.

7. (a) Suppose \( F(x) \) is the cumulative distribution function for heights (in meters) for trees in a forest.

i. Explain in terms of trees the meaning of the statement \( F(7) = .6 \).

ii. Which is greater, \( F(7) \) or \( F(6) \)? Justify your answer in terms of trees.

(b) Suppose \( p(x) \) is the density function for lengths of lumber used in building a house. What is the meaning of the statement \( p(96) = 0.35 \)?

**Sol:**

(a) i. The probability that a tree chosen at random has height at most 7m is .6.

ii. \( F(7) \geq F(6) \) since the probability that the height is at most 7m must be at least as big as the prob the height is at most 6m.

(b) Approximately 35 percent of boards used to build the house have lengths from 95.5 to 96.5.

8. A ball is dropped from the height of 15 ft. Each time it hits the ground, it rebounds to \( 4/5 \) of the height it attained on the previous bounce. Assuming that this process goes on forever, find the total distance the ball travels.

**Sol:**

\[ 15 + 2 \cdot 15 \cdot \frac{4}{5} + 2 \cdot 15 \cdot \left( \frac{4}{5} \right)^2 + 2 \cdot 15 \cdot \left( \frac{4}{5} \right)^3 + \cdots \]
\[
\frac{\sum_{k=0}^{\infty} 30 \left(\frac{4}{5}\right)^k - 15}{1 - 4/5} - 15 = 135
\]

9. Consider the function \( p(x) \) defined by \( p(x) = \frac{x^2}{9} \) for \( p < x < 3 \) and \( p(x) = 0 \) otherwise.

(a) Show that \( p(x) \) is a probability density function.
(b) What proportion of \( x \)'s have values between 1.2 and 1.8.
(c) Compute the mean of this distribution.

Sol:
(a) \( p(x) \geq 0 \) for all \( x \) and
\[
\int_{0}^{3} \frac{x^2}{9} \, dx = 1
\]
(b)
\[
\int_{1.2}^{1.8} \frac{x^2}{9} \, dx = .152
\]
(c)
\[
\int_{0}^{3} \frac{x^3}{9} \, dx = 2.25
\]

10. Does the series
\[
\sum_{n=0}^{\infty} \frac{2^n}{n^3 + 1}
\]
converge or diverge? Justify your answer and show the work associated with any test you use.

Sol: Use the ratio test.
\[
\frac{|a_{n+1}|}{|a_n|} = \frac{2(n^3 + 1)}{n^3 + 3n^2 + 3n}
\]
and the limit as \( n \to \infty \) of the above quotient is 2, which is bigger than 1. So by the ratio test, it diverges.