Be careful to **show your work** and label all graphs.

**The answers are given at the end.**

(1) (a) Give a formula that represents the average value of

\[ f(x) = \frac{10}{x^2} \]

from \(1 \leq x \leq 3\).

(b) Evaluate the formula given in part (a) using the Fundamental Theorem of Calculus.

(2) The function \(f(x)\), given in Figure 1 represents the rate at which bikes are sold in bikes per day at a given bike shop over a 1 year period. Here, \(x\) is the number of days since Dec 31.

(a) Express as a definite integral the average number of bikes sold in June.

(b) Approximate the value of the expression given in part a.).

(c) Find the average number of bikes sold from the beginning of February through the end of April.

(3) A bar of metal is cooling from 1000 degrees Celcius to room temperature, 20 degrees Celcius. The temperature, \(H\), of the bar \(t\) minutes after it starts cooling is given by

\[ H = 20 + 980e^{-0.1t}. \]

(a) Write a formula for the average value of the temperature of the bar during the first 3 hours.

(b) Find the average value using the Fundamental Theorem, check by using your graphing calculator.
(4) A town has a population of 1000. In each of the following cases, write a formula for each and fill in the table assuming that the town’s population grows by

(a) 50 people per year

<table>
<thead>
<tr>
<th>Year</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population</td>
<td>1000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) 5% per year

<table>
<thead>
<tr>
<th>Year</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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</thead>
<tbody>
<tr>
<td>Population</td>
<td>1000</td>
<td></td>
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</tbody>
</table>

(5) Suppose the population of fleas is a given function $P = r(t)$ in thousands per day and has an absolute growth rate of $\frac{dP}{dt} = 60P$. Find the change in population from $t = 0$ to $t = 10$ hours.

(6) Find the indefinite integral

$$\int (3t^4 - \frac{2}{t^2} + \cos t - e^{2t} + 3) \, dt$$

(7) The rate, $r$ at which people within a certain university get sick during an epidemic of the flu can be approximated by

$$r(t) = \frac{20t}{t^2 + 1},$$

where $t$ is measured in days and $1 \leq t \leq 60$. Find the total quantity of students who came down with the flue during its entire course. (60 days)

(8) The graph of $g'(x)$ is given in Figure 2. Approximate the values of $g(x)$ for $x = 3, 5$ given that $g(0) = 1$.

(9) A formula that represents the function $f(t)$ is given.

$$f(t) = \frac{2t}{t^2 + 7}$$
(a) Find the indefinite integral
\[ \int f(t) \, dt \]
(b) Use your answer to part (a) to find
\[ \int_1^2 f(t) \, dt. \]

(10) A formula that represents the function \( f(t) \) is given.
\[ g(z) = 60e^{-0.01z} \]

(a) Find the indefinite integral
\[ \int g(z) \, dz \]
(b) Use your answer to part (a) to find
\[ \int_0^1 g(z) \, dz. \]

Solutions

(1) (a) \( \frac{10}{3} \int_1^3 x^{-2} \, dx \)
(b) \( \frac{10}{3} \)

(2) (a) \( \frac{1}{30} \int_{152}^{182} f(x) \, dx \)
(b) 100
(c) 165

(3) (a) \( \frac{1}{180} \int_0^{180} 20 + 980e^{-0.1t} \, dt \)
(b) 66.4433

(4) (a) 1050, 1100, 1150, 1200
(b) 1052, 1106, 1162, 1222

(5)

(6)

(7)

(8) \( g(3) = -23, \ g(5) = 37. \)

(9)

(10)